Virginia's Mathematics Specialist Initiative:
Overview of Program and Course Annotated Syllabi for Preparing Mathematics Specialists

The Virginia Mathematics and Science Coalition

May 25, 2016

Compiler's Note

The information contained in this report has been compiled from the archives of the Virginia Mathematics Specialist Initiative (VMSI) program development, publications, and curriculum development work. A large number of members of the mathematics community contributed to its creation. Support from the National Science Foundation is acknowledged and very much appreciated. The steering committee for VMSI takes sole responsibility for the content of this report.

Correspondence concerning this report should be addressed to
Vickie Inge at vickieinge@gmail.com or Bill Haver at wehaver@vcu.com.

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Executive Summary

For more than 20 years the Virginia Mathematics and Science Coalition (VMSC) and the mathematics and mathematics education community have championed the Virginia Mathematics Specialist Initiative (VMSI) and the potential for K-8 school-based mathematics specialists, in-school coaches who support teachers, to improve student learning. The VMSC with support from a grant award through the State Council for Higher Education in Virginia (SCHEV) and several National Science Foundation (NSF) grants has supported statewide collaborations to develop answers to the following questions (Pitt, 2005, 24).

- What is a Mathematics Specialist and what do Specialists do?
- What are the principle ingredients of content, content pedagogy, and leadership training Mathematics Specialists will need to be effective?
- How can we implement Mathematics Specialist training programs and quickly bring well-prepared Mathematics Specialists into the schools?
- What are the elements of school culture and administrative support that Mathematics Specialist programs need to be effective?

The report, *Virginia's Mathematics Specialist Initiative: Overview of Program and Course Annotated Syllabi for Preparing Mathematics Specialists*, has been prepared by the VMSI steering committee for the VMSC. The document is informed, in particular, by the knowledge gained and products developed as a result of the NSF grant projects that supported preparing and launching school-based mathematics specialists. The report first presents an overview of the VMSI and then a general description of the Virginia Mathematics Specialists Preparation Program (VMSPP), and finally, an annotated syllabus for each of six mathematics content courses and three mathematics education leadership courses developed and taught as the core of the VMSPP.

The paper describes the SCHEV and NSF grant supported efforts; and also, the ExxonMobil Foundation support is described since the Foundations was crucial for the early efforts of VMSC to inform various audiences about the potential of improving mathematics education through the support of school-based mathematics specialists. The various audiences included members of the K-16 mathematics and mathematics education community, professional organizations such as the Virginia Council of Mathematics Specialists (VACMS), Virginia Council of Mathematics Teachers (VCTM) and the Virginia Council of Mathematics Supervisors (VCMS), and the mathematics leaders in school divisions across the state.

The partnerships that emerged during the activities supported by ExxonMobil provided a foundation of knowledge and support for successful grant proposals for NSF and Virginia's State Council of Higher Education (SCHEV) awards. The awards supported statewide collaborations including a comprehensive project, the Virginia Mathematics Specialist Initiative (VMSI). The activities of the VMSI under the umbrella of the VMSC paved the way for the *Virginia Licensure Regulations for School Personnel* to add the K-8 Mathematics Specialist endorsement. Furthermore, VMSI provided leadership for identifying a program and creating core courses to prepare successful classroom teachers to seek the licensure endorsement and to assume the role of a K-8 school-based mathematics specialist. Significant collaborations among institutes of higher education (IHE), school division mathematics leaders, and state organizations dedicated to improving mathematics education are described in this report along with the efforts that clarified the vision for the knowledge and skills that mathematics specialists require to fulfill their various roles successfully. The report concludes by describing the VMSPP and the core mathematics and leadership courses included in the Program.

The information presented in this report has been compiled by reviewing and extracting information found in various curriculum projects, reports, and journal publications resulting from activities in the grant-supported Virginia Mathematics Specialist Program produced by the numerous collaborators in VMSI project over the last ten years. The contributions of the following people filled key roles as curriculum development committee members, higher education instructors, school division co-instructors, researchers, and grant management committee members have been instrumental in developing this report. Please note that the list presented in Figure 1 is not an exhaustive list of the many people who also made important contributions to the program including those school divisions, mathematics specialists, and teachers who graciously agreed to participate as research subjects.

Figure 1. Grant Projects Participants.

Participant	**Organization Affliliation**[1]
Robert Berry	University of Virginia
David Blount	Virginia Commonwealth University
Patricia Campbell	University of Maryland
Pam Cropp	Culpeper Public Schools
Debra Delozier	Stafford Public Schools
Aimee Ellington	Virginia Commonwealth University
Rhonda Ellis	Norfolk State University
Reuben Farley	Virginia Commonwealth University
Ena Gross	Virginia Commonwealth University
William (Bill) Haver	Virginia Commonwealth University
Vandi Hodges	Hanover Public Schools
Jeff Holt	University of Virginia
Vickie Inge	University of Virginia
Jamey Lovin	Chesapeake Public Schools
Phillip McNeil	Norfolk State University
Megan Murray	University of Virginia
Sandra Overcash	Virginia Beach Public Schools
Loren Pitt	University of Virginia
Patricia Robertson	Arlington Public Schools
Sean Smith	Horizons Research Inc.

Judy Singleton	Virginia Commonwealth University
Sharon Emerson-Stonnell	Longwood University
Dewey Taylor	Virginia Commonwealth University
Christine Trinter	Virginia Commonwealth University
Denise Walston	Norfolk Public Schools
Joy Whitenack	Virginia Commonwealth University
Melvin (Skip) Wilson	Virginia Polytechnic Institute and State University

1. Primary organization affiliation during the grant projects.

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Section 1:

The Virginia Mathematics Specialist Initiative

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The Virginia Mathematics Specialist Initiative

The VMSI is a multifaceted collaborative effort involving more than 50 school systems, 10 universities, the VDOE, three Virginia mathematics education organizations, and the VMSC. Before describing the core mathematics content and education leadership courses that are included in the VMSPP, a description of the early collaborations includes information about grant funding that provided support for activities in the VMSI, the working definition of a school-based mathematics specialist, and policy implications. Support from four National Science Foundation (NSF) grants and an Eisenhower grant through the State Council for Higher Education in Virginia (SCHEV) and with leadership support from the VMSC six Virginia universities collaboratively developed core course work as part of a cooperative master's degree programs to prepare individuals to serve as mathematics specialists. Carefully designed scientific treatment/control research studies supported by NSF grants have demonstrated that mathematics specialists make a substantial, statistically significant difference in student achievement and teacher beliefs.

Information about Mathematics Specialists and research results can be retrieved from the internet at the VMSC web page at http://www.vamsc.org/. In addition The VMSC's *The Journal of Mathematics and Science: Collaborative Explorations* has published a number of special editions that focus on the work of Specialists. The journals document much of the grant work and results from researchers Campbell, Horizons, and Whitenack and Ellington. In addition, articles written by Blount and Singleton address the public policy issues and feedback from school division leaders including principals. Copies of these focus journals can be retrieved from http://math.vcu.edu/outreach/the-journal-of-mathematics-and-science-collaborative-explorations/.

Early Collaborations Lead to a Vision

A significant early collaboration paving the way for the Virginia Mathematics Specialist Initiative (VMSI) began in 1992 when the Virginia Mathematics Coalition, now the Virginia Mathematics and Science Coalition (VMSC), joined with the Virginia Department of Education (VDOE), the Virginia Council of Teachers of Mathematics (VCTM), the Virginia Council for Mathematics Supervision (VCMS) and multiple universities in an NSF-funded project, V-QUEST. The project proposed to prepare elementary and middle school classroom teachers to serve as "Math Leaders" or "Science Leaders" in their schools. Over the three years of funding, participating K-8 mathematics and science teachers increased their knowledge in mathematics and science content and in content pedagogy during intensive and focused summer institutes. These regular classroom teachers assumed add-on duties, and they did provide some helpful leadership before and after school. However, the time allowed to a regular classroom teacher for supporting other teachers was not sufficient to support teachers in changing mathematics instruction to improve student learning. Schools needed a mathematics specialist or coach without classroom assignments which could work with strengthening the mathematics instruction of all teachers.

Another important event allowing for collaboration took place May 20, 2001, a forum, "Moving from Teacher Leaders to Mathematics Teacher Specialists" took place in Fredericksburg, Virginia. This forum was the first of three significant forums hosted by VMSC with support from ExxonMobil to bring together various stakeholders from across the state to gather information and provide an opportunity for discussion about the potential value of mathematics specialists as well as what preparation would be necessary to fulfill the roles envisioned. Participants in the first forum included representatives from the VMSC, VCMS, VCTM, university mathematicians and mathematics educators, school division leaders, and the VDOE Assistant Superintendent of Instruction. The forum participants were in agreement; a *well-prepared* mathematics teacher specialist could be an effective support for classroom teachers.

As a result of the Fredericksburg forum, the VMSC appointed a task force, in 2002, to review the literature and learn more about how a mathematics specialist embedded in a school might improve instruction and consequently student learning. In addition, the task force was charged with making recommendations for the potential roles and responsibilities a specialist might assume and about the preparation to become an effective mathematics specialist. The Mathematics Specialist Task Force Report (VMSC Task Force, 2005, p. 3) put forward a strong recommendation that *well-prepared mathematics specialists* should be placed in elementary and middle schools to help teachers strengthen their mathematics knowledge for teaching along with their instructional practices for teaching mathematics so that every Virginia student could reach high levels of mathematics achievement. In addition, the Task Force Report made specific recommendations about the necessary knowledge and skills and for the mathematics content and leadership experiences to be included in the preparation program.

Grants Supporting the Virginia Mathematics Specialist Initiative

The following activities were supported by ExxonMobil.

- Beginning in the late 1990s, individual school systems secured grants through the ExxonMobil Elementary Mathematics Leadership Program to prepare teacher leaders. The school divisions included Stafford, Hanover, Bedford, Arlington, Alexandria, and Prince William.
- Three statewide networking forum grants awarded to VMSC in 2001, 2003, and 2004 to build capacity for Mathematics Specialists (State Superintendent of Education, Patricia Wright spoke at one of the forums, President of the National Council of Teachers, Skip Fennel spoke at another, and Governor Mark Warner visited another).
- Publication of special issues of the VCMS *Journal of Mathematics and Science: Collaborative Explorations,* sharing information nationally on the impact of Specialists.
- Making available the extremely important personal support of Patrick Dexter, who served on VMSC Advisory Boards, visited schools, and testified before legislative bodies.

Beginning in summer 2002, under the leadership of VMSC 2002, a sequence of three Virginia Mathematics and Science Partnership (MSP) grant awards through the state flow-through funds involved forty-five Virginia school divisions multiple universities. The grants supported the first efforts in developing and offering mathematics and mathematics education leadership courses specifically designed to prepare elementary mathematics specialists. Continuing the efforts to design, teach, and refine the core coursework for a mathematics specialist preparation program was supported by a series of four five-year NSF projects. The projects fell under the VMSC umbrella and involved collaborations among Virginia Commonwealth University (VCU), University of Virginia (UVA), Norfolk State University (NSU), Longwood University (LU), and University of Maryland. Research carried out as part of the NSF grant projects confirms the positive benefits of having a well-prepared mathematics specialist working with teachers in a school to improve student achievement. This research can be accessed at http://www.vamsc.org/.

1. (ESI-0353360) Mathematics Specialist in K-5 Schools: Research and Policy Pilot Study (6/1/04 - 5/31/10)
2. (DUE-0412324) MSP Preparing Virginia's Mathematics Specialists (8/1/04-7/31/13 which included several supplements)
3. (DRL-0918223) Research the Expansion of K-5 Mathematics Specialist Program into Rural School Systems (9/1/09 – 8/31/15, with the no-cost extension)
4. (DUE-0926537) MSP Institute: Mathematics Specialists in Middle Schools (8/1/09 – 7/31/15, with the no-cost extension)

VMSI Model Mathematics Specialist Preparation Program

The VMSI promoted school-based mathematics specialists to support teachers in Virginia elementary and then middle schools with three notable successes: 1) Virginia established a mathematics specialist endorsement for elementary and middle education, 2) twelve state universities established master's degree programs to prepare mathematics specialists, and 3) the Virginia Board of Education recommended the placement of one specialist in schools for every 1,000 students. Though the recommendation is currently unfunded, school districts have been creative in using state funds such as Algebra Readiness Funds, and local sources to create specialist positions.

The work of the VMSI over more than 10 years has lead to a model for a VMSPP (VMSPP). Following the 2002 Task Force, with the support of a SCHEV MSP grants and the first two NSF grants, five mathematics content courses and three mathematics education leadership courses were specifically created to provide mathematics specialists with strong school content knowledge and important content pedagogical and leadership knowledge and skills. In the beginning, the courses were focused on the development of elementary mathematics specialists. Well prepared specialists were placed in the treatment schools to determine the effectiveness of a school-based mathematics specialist providing on-site coaching for teachers when compared to a control school. Findings revealed that well prepared elementary mathematics specialists working with teachers were making a difference (Campbell & Malkus, 2011). Over time, classroom teachers who worked with a mathematics specialist developed more effective teaching practices that positively affected student learning and achievement. In addition principals in the treatment schools reported that they found the specialist to be an important resource in their building (Blount & Singleton, 2007) and interviews with central office leaders in the participating school divisions revealed their support of mathematics specialists and the important role they played in the elementary school buildings (Blount & Singleton, 2008).

Leaders in VMSI realized there was a gap between the *K-5 preparation program* and the *K-8 Mathematics Specialist licensure endorsement*. Much had attention had been given to preparing elementary school specialist and that additional attention was to put a program in place to have well-prepared middle school specialists. Consequently, the VMSC formed the 2008 Middle School Mathematics Specialists Task Force to consider how the K-5 Mathematics Specialist Preparation Program could be modified to address needs of middle school mathematics specialists who would, in turn, effectively support middle school mathematics teachers. As with the 2002 Task Force, the participants included university mathematicians, mathematics educators, and district mathematics supervisors. In addition, practicing mathematics specialists were invited to participate. The Task Force considered the unique demands relevant to the work of a middle school specialist. The group recognized that the middle school specialist must accommodate factors that were more prevalent in elementary than in middle school by the nature of the number of years the student has been in school and the organizational structure of middle schools. Middle school specialists support teachers who students with a wider range of academic needs, call upon a range of skills to manage scheduling and organizational constraints, find ways to help teachers motivate students who lack confidence in mathematics and manage more autonomy in fulfilling their role. Building on the school-based mathematics specialist responsibilities identified by the 2002 Elementary Mathematics Specialist Task Force, the Middle School Task Force, 2008, presented the following definition of a mathematics specialist and the responsibilities a specialist may be expected to assume in a middle school (VMSC, 2009, 17).

Recommended School-based Mathematics Specialist Responsibilities

> Mathematics Specialists are teacher leaders with strong preparation and background in mathematics content, instructional strategies, and school leadership. Based in elementary and middle schools, Mathematics Specialist are experienced teachers who are released from full-time classroom responsibilities so that they can support the professional growth of their colleagues, promoting enhanced mathematics instruction and student learning throughout their schools, They are responsible for strengthening classroom teachers' understanding of mathematics content, and helping teachers develop more effective mathematics teaching practice that allows all students to reach high standards, as well as sharing research addressing how students learn mathematics.
>
> The overarching purpose of the mathematics specialists is to increase the mathematics achievement of all students in their schools. To do so, they:
>
> - Collaborate with individual teachers, teams of grade level mathematics teachers, and with vertical teams across grade levels through co-planning, co-teaching, and coaching;
> - Assist administrative and instructional staff in interpreting data (both formative and summative) and designing approaches to improve student achievement and instruction;
> - Collaborate with teachers and teams of teachers to ensure that the school's

instructional practices are aligned with state and national standards, as well as their school division's mathematics curriculum;
- Assist teachers' with delivery and understanding of the school curriculum through collaborative long-range and short-range planning;
- Facilitate teachers' use of successful, research-based instructional strategies, including differentiated instruction for diverse learners, and appropriate use of technology;
- Provide job-embedded professional development focused on both mathematical content knowledge and mathematical pedagogy;
- Assist teachers in fostering partnerships with parents/guardians and community leaders to foster continuing home/school/community relationships focused on students' learning of mathematics; and,
- Collaborate with administrators (both in and outside of the mathematics community) to develop a vision and to provide leadership for the professional development and for a school-wide mathematics program.

Following the Middle School Specialist Task Force Report, teams of educators collaborated to add specific middle school content to the existing content and leadership courses. An additional algebra course was designed for middle school specialists; stronger connections were added between the algebra courses and the numbers and operations and rational numbers courses. The geometry course incorporated high school geometry concepts and the use of dynamic geometry software, and the probability and statistics course added more analysis of data techniques. Course assignments were revamped to differentiate for participant school level placement, and the leadership courses included issues specific to middle school in terms of equity, interdisciplinary teaming and co-teaching. Other challenges such as addressing wider learning gaps and tackling student motivation issues at the middle school level were also incorporated.

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Section 2:

Overview of the Virginia Mathematics Specialist Preparation Program

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Overview of the Virginia Mathematics Specialist Preparation Program

Twelve Virginia universities currently offer a master’s degree program to prepare mathematics specialists, core mathematics, and leadership courses make up the programs in addition to some unique set of program expectations defined by the university. What follows is a description of the core courses as developed and piloted with support through the series of four five-year NSF projects, listed previously, during the collaborative efforts of Virginia Commonwealth University (VCU), University of Virginia (UVA), Norfolk State University (NSU), and Longwood University (LU) under the VMSC Statewide Master Degree Programs initiative (VMSC, n. d.).

Mathematics Content and Leadership Courses

From the beginning, designers of the VMSPP realized that teaching courses to prepare teachers for a mathematics specialist role would be unique; instructors would be teaching the coaches of the mathematics teachers. The 2002 Task Force Report highlighted, modifying existing college mathematics courses such as number theory, geometry, or algebra would not meet the needs of mathematics specialists (VMSC, 2005, p. 16). New 3-hour graduate level mathematics and mathematics education leadership courses needed to be created. Courses were needed that connected mathematics content knowledge to content pedagogical knowledge and that allowed teachers to understand the developmental progression of mathematical ideas necessary for planning instruction and assessment. In addition, these future leaders would need to recognize how the information from their coursework bridged to their own teaching practices and how information from their coursework would be reflected in the coaching practice. The Middle School Mathematics Specialist Task Force (2008) supported the recommendations made by the 2002 Task Force and reiterated that, "helping participants recognize how assignments from their coursework translated into their practice both as teachers and as coaches is a critical obligation of the curriculum and the course instructors" (VMSC, n. d., p. 16-18).

A brief description of six mathematics content and three mathematics education leadership courses follow. An annotated syllabus for each of these courses is located in the following section of this report.

Numbers and operations. This introductory course addresses fundamental mathematical ideas concerning the operations of arithmetic and the base-ten number system. Connections between the operations are explored in various contexts including whole numbers, problem solving, decimals, and fractions. The structure of the number system is used to develop understandings of our base-ten system. The course also uses cases about students’ thinking and the computational methods they use and episodes in the history of the number system that illuminate the developmental progression of the mathematics and the learning trajectories of children.

Rational numbers and proportional reasoning. In this course, students explore the conceptual and procedural basis of rational numbers; fractions, decimals, and percents as well as the essential role that proportional reasoning plays in mathematics. The logic, estimations, interpretations, and procedures used when ordering and computing with fractions and decimals are explored using multiple interpretations and representations including visual and physical representations. Episodes from the history of the number systems are explored and compared with the developmental sequence and learning trajectories of children learning this material.

Algebra and functions. Students develop skills in representation, generalization, and development of mathematical arguments through the exploration of the properties of arithmetic operations, the relationship between operations and operating on particular numbers. Additional topics from algebra that are explored are variables, patterns, and functions; modeling and interpretations of graphs; linear functions and non-linear functions, including quadratics and exponentials.

Algebra for middle school specialists. This course extends the understanding of topics introduced in the Functions and Algebra I course, introduces new topics from secondary mathematics, and integrates graphing technology into the study of the algebra topics. Class activities focus on extending students' skills in representation, generalization, and developing mathematical arguments. Topics include but are not limited to linear equations and inequalities; modeling and interpreting graphs; linear and non-linear functions; logarithms; factoring, zeros, and intercepts; domain and range; exponents and radicals; and some number theory related to the real number system.

Geometry and measurement. This course explores the foundations of informal geometry and measurement in 1, 2, and 3 dimensions. The van Hiele model for geometric learning is used as a framework to explore how children build their understandings of length, area, volume, angles, and geometric relationships. Visualization, spatial reasoning, and geometric modeling are stressed along with transformational geometry, congruence, and similarity.

Probability and statistics. Various elementary statistical measures and graphical representations are used to describe, compare, and interpret data sets. The basic laws and concepts of probability are explored including sample spaces, probability distributions, and random variables. A statistical project is required that uses hypothesizing, experimental design, the collection of data, and comparisons of different populations.

Leadership I. This introductory course is designed to build an understanding of the content and process standards identified by the National Council of Teachers of Mathematics (NCTM) *Principals and Standards for School Mathematics* (2000) and the K-8 *Virginia Mathematics Standards of Learning and Curriculum Framework*. In addition, connections are made within the mathematics content as participants develop their knowledge about mathematics, mathematics content pedagogy and diagnosing student understanding. A focus is given to students as mathematics learners with attention to learning theory, formative assessment, and diverse learners; teachers as learners through study groups and observation of another teacher’s classroom; and the instructional program through the design, teaching, and evaluation of student-centered lessons.

Leadership II. This course is designed to build skills, understandings, and dispositions required for optimal mathematics education leadership roles in K-8 schools; in particular the different roles of the school-based mathematics specialist. The course develops skills to coach and work with adult learners, understanding mathematics content pedagogy necessary to support teachers, using research in selected topics for instructional decision making, and building deeper understandings of the mathematics that underpins the K-8 mathematics curriculum.

Leadership III. This course builds skills, understandings, and dispositions required for optimal mathematics education leadership roles in K-8 schools; attention is given to data analysis and collaborative data-driven discussions for instructional planning and for mathematics program decision making. In addition, students engage in learning to participate in and to facilitate the Lesson Study process; to develop and use formal and informal formative assessments to guide instruction; to develop or modify tasks for effective task-based mathematics instruction, and to support other teachers effective mathematics lesson planning.

Considerations when Planning a Course

Courses in the VMSPP have been taught in various formats, and each format presents different advantages and challenges to the students and the instructors. As part of the VMSI, courses have been offered in residential summer institutes with about 55 hours of class time and significant daily in class and homework assignments including readings, doing mathematics, and writing reflection papers. As traditional semester classes, taught in 15 three-hour weekly sessions each with homework assignments including readings, doing mathematics, writing reflection papers, and writing cases. A third option, more often used with the leadership courses was to split the time between summer sessions and Saturday classes.

Summer residential institute format. Students who participated in a program offered entirely as residential institutes took courses each of three consecutive summers. Content courses were taught simultaneously during the first summer institute so that on a given day participants experienced one course in the morning and another in the afternoon for the five weeks. Feedback on this schedule was not as positive, so the schedule was adjusted to have one course follow the next. In the following summer institutes, two content courses were offered in succession over a five week period. Specifically, the two content courses were offered in intense 2½-week sessions designed for two 3 ½ hour blocks per day.

In addition, to the two content courses each summer, the first half of a leadership course was also scheduled, held four times spread out over the five-week institute. The second half of the leadership course was held on four Saturdays spread throughout the fall semester for 6 hours each time. This allowed participants to work with students and teachers in their schools when completing class projects.

Semester format. Students enrolled in a semester content course met one night a week for 3 hours. During the summer, the course was taught either for two full weeks or for two or three days spread over several weeks. Leadership courses were generally offered in the fall semester sometimes overlapping into the second semester which gave participants more time to work with students and teachers in completing class projects.

Blended learning format. The core courses in the Mathematics Specialist Program were originally designed to be face- to-face courses. However, in reaching out to rural school districts across Virginia in one of the NSF grants, *Research the Expansion of K-5 Mathematics Specialist Program into Rural School Systems*, it was evident that travel to class would be a major obstacle to teachers who wanted to participate. In an effort to address the travel challenge or teachers, the program was offered in a combination of the hybrid formats; blended and residential summer institutes as well as blended semester courses.

Technology allowed the face-to-face courses to be repurposed to fit a blended format. To maintain the cohesiveness of a cohort, the blended courses met twice for a 2-day, Friday and Saturday, face-to-face meeting, at the beginning and then midway through the course, with the remainder of the classes meeting synchronously online. The sequence of courses remained similar to the original design with a few exceptions. Each cohort began with the *Number and Operations* course. The use of Digimemo L2, while not without challenges, in conjunction with a universities online collaboration platform allowed students to share work and to participate in small group chat rooms. This allowed online classes to be dynamic and interactive. The online classes used whole-group and small-group real-time discussions in class. Group projects were assigned with the expectation that students would use the online infrastructure to meet with their groups online. Formative evaluation reports from Horizon Inc. made to the course development teams about the change in students' knowledge as measured on pre- and post-course assessments in the blended courses revealed no significant difference from students who participated in the face-to-face course.

The cohort program for the K-5 Mathematics Specialist Program into Rural School Systems grant included three face-to-face mathematics content classes, and three other core content classes were offered as blended classes. The face-to-face class offered the first summer was *Geometry and Measurement*; the second summer was *Algebra and Functions,* and the third summer was *Mathematics for Diverse Populations*. An additional course was taught in the blended format each of the first two summers; *Rational Numbers and Proportional Reasoning* and *Probability and Statistics*. All three leadership courses were taught in a blended format during the school year. *Leadership I* and *II* were taught the fall and spring semester following the first and second summers respectively and *Leadership III* followed the third summer institute during the fall semester.

The format in which a course is offered impacts participant experiences in different ways. In the summer residential institute format, students are immersed in the work and have the opportunity for additional collaboration with their peers after class hours. Participants do not, however, have the opportunity, as they do in the school year semester format, to do the mathematics with their own students, to interview students about their understanding of the mathematics, or write their own case studies. During the school-year format, there may be more time between classes for reflection and making connections than is readily available during the residential institutes when classes meet all day on consecutive days. Some participants reported, however, that the residential institutes allowed them to focus on the coursework, and they were not interrupted by the daily demands of home or work. Time can become an issue in any format, so careful planning and pacing are essential.

Course sequencing. Figure 2 shows the recommended sequence of content courses with the introductory course, *Number and Operations*, because it provides the foundation for the philosophy and pedagogical methodology of the mathematics specialist preparation program. Then followed by the *Rational Number and Proportional Reasoning* course. During the summer institute program, these were offered the first summer overlapped by the *Leadership I* course which finished at the end of the fall semester. In a semester schedule, *Number and Operations* were offered in the fall semester, and the *Rational Number* course in the spring. *Leadership I* began in September and continued into the second semester ending in February.

Figure 2. Course Sequencing.

Mathematics Content Courses	Mathematics Education Leadership Courses
Number and Operations (Required first course)	Leadership I
Rational Number and Proportional Reasoning	
Geometry and Measurement	Leadership II
Algebra and Functions	
Probability and Statistics	Leadership III
Algebra for Middle School Math Specialists	

The second year of a summer institute format included the *Geometry and Measurement* course and the *Algebra and Functions I* course along with the first half of *Leadership II* which concluded at the end of the fall semester. In a semester schedule, *Geometry and Measurement* were offered in the fall, followed by the *Algebra and Functions* course in the spring, with the *Leadership II c*ourse running from September through February.

The third year of a summer institute program includes *Probability and Statistics* followed by *Algebra for Middle School Specialists*, or a diverse learners course was offered if only elementary specialists were involved. *Leadership III* began in the summer and concluded at the end of the fall semester. In the semester schedule, *Probability and Statistics* were offered in the fall along with the *Leadership III* course, and then *Algebra for Middle School Specialists* in the spring. The *Leadership III* course concluded at the end of the fall semester to allow students to concentrate on their degree-granting university practicum or externship requirements during the last semester of the program.

Student cohort model. When possible the program should be offered to a group of teachers who are committed to completing the required work for the entire VMSPP. The support that teachers offer one another in a cohort is essential, especially as the course work increases in content difficulty. The courses are designed to include, productive mathematical discourse, project-based learning opportunities, and collaborative in-class group work. Therefore, a cohort provides an opportunity for the students to develop bond around a learning community. The strong bond that develops over time extends beyond the program in professional networks as the teachers assume roles as mathematics specialists.

Factors Impacting Optimal Instruction

The mathematics courses carried graduate mathematics credit. Because of the unique preparation necessary in mathematics for teaching necessary for a mathematics specialist, the specialist courses were taught by instructor teams comprised of two and sometimes three instructors with different backgrounds and experiences as mathematicians, mathematics educators, or school division mathematics leader or specialist. The teaching team worked collaboratively to analyze the mathematics and also to connect the mathematics to teacher practice. Planning, teaching, and assessment were all cooperatively done as instructors modeled co-teaching and research informed effective teaching practices.

The leadership courses carried mathematics education graduate credit, so it was critical that a mathematics education instructor or adjunct instructor from an institute of higher learning be on the team. Other members of this team included a K-12 mathematics supervisor and an experienced mathematics specialist. Participants analyzed teaching in their own classrooms, learning to coach one-on-one and in small groups, and learning how to impact best the mathematics program at their school. It was necessary that at least one of the instructors have experience in a K-12 school setting working with teachers and administrators to support a strong mathematics program. Each member of the teaching team brought an expertise to the team for collaboratively planning instruction and teaching and assessing students. In addition, the instructors and the students were each experienced professionals with much to offer to the learning environment and much to learn from each other.

Prior to the start of each summer institute, the program leaders provided an opportunity for the course instructors to come together to identify goals, review any data from the outside evaluators and from student observations and course exit slips, and to coordinate their work within and between courses. In particular, instructional strategies were discussed. Instructors met again as students transitioned from one summer course to the next to identify potential connections that could be made among courses. It was particularly helpful for the leadership course instructors to hear from the content course instructors as they planned for the projects in the leadership courses. Numerous instructors were involved in more than one course during each cohort's program as well in different cohort's program, and this continuity was helpful in maintaining consistent expectation from course to course as well as from cohort to cohort and also in continually improving the learning experience for students. It was also beneficial to bring new instructors to first work with veteran instructors to learn the philosophy of the program and to gain experience teaching a course for mathematics specialist.

Instructors were sensitive to the demands on the students who were trying to juggle their course work and full-time teaching responsibilities as well as family needs. Instructors from one course communicated with instructors of previous courses in the program sequence, so they were fully versed in the prior knowledge held by the group. If all students have participated in the same cohort, this background knowledge can be more accurately defined. Program leaders provided an opportunity to bring instructors together to review course content and check on student progress.

Regardless of the scheduling or formatting of courses the Virginia Mathematics Specialist Program remained focused on providing the foundation strong teacher leaders needed to successfully transition from the classroom to becoming mathematics specialists with responsibilities for coaching other teachers.

Teaching Coaches of Mathematics Teachers

The instructors in the VMSPP were committed to preparing strong classroom teachers with at least three years of teaching experience with the knowledge, skills, and dispositions that are particular to the leadership they would provide in their new role as mathematics specialists and coaches of teachers. The 2002 VMSC Mathematics Specialists Task Force identified key responsibilities of elementary mathematics specialists (VMSC, 2005, p.15) and the VMSC Middle School Mathematics Specialists Task Force refined the definition of for middle school mathematics specialists (VMSC, 2009. p. 17) and identified what middle school mathematics specialist should know and be able to do. Moving from being an experienced, successful classroom teacher to being a novice mathematics coach is challenging. Mathematics specialists are coaches for their fellow teachers, supporting them to be more effective in planning instruction to grow, teaching, and assessing conceptual mathematical knowledge and procedural skills. Much thought and research went into the development of each of the core courses as well as the careful fusion of these courses into a program that will provide an effective and cohesive pathway for preparing successful teachers to become successful mathematics specialists. Instructors must be intentional about considering what it means to teach coaches of teachers as they facilitate the learning in order to develop mathematics specialists' leadership skills.

The 2002 VMSC Mathematics Specialists Task Force Report recommended ways that mathematics specialists should be prepared for the leadership roles they would assume in schools (VMSC, 2005, p. 18-19). The task force called for courses designed to leadership skills include a focus on building perspective specialists' deep understanding of *how* students learn and make sense of mathematics and on pedagogical knowledge specific to mathematics teaching and learning. In order to support a school's mathematics program and its teachers, the leadership courses developed the future mathematics specialists knowledge and skills to identify and use curriculum based on current research, including national and state standards for mathematics, and to design and to support instruction to address the needs of diverse learners. In addition the leadership courses develop knowledge and skills necessary to collaborate with the principal to analyze the impact of the school's mathematics program on student learning, to identify areas in need of strengthening, and to identify ways to address the identified instructional problems.

Course work in the program enabled students to gain skills in analyzing individual student performance on a variety of formative assessment protocols, and in analyzing and interpreting individual as well as collective assessment data from formative and summative assessments. Students learned about data-driven decision making to inform instructional decisions. In addition, students learned to gather and interpret relevant data about instruction in regards to student learning and instructional programs to facilitate improvements in student learning. (VMSC, 2005, p. 18-19)**.**

Careful attention was given to preparing mathematics specialists to work as a collaborative leader with all school stakeholders; administrators, teachers, parents, and the community. The ability to work with adult learners in order to help build a strong and effective mathematics community within the school was a critical component of all coursework. Being able to identify ways to improve student achievement, communicating the what needs to change in a tactful and positive manner and working collaboratively to implement steps to meet these needs, are leadership skills each developed during the program. Effective written and oral communication skills were expected throughout the program. In order for students to exit the program as well-prepared mathematics specialists, the program had high expectations that the instructors would actively engage the participants in group work and in simulations about leading activities in class and in their schools.

Instructors model proven effective practices. Instructors were intentional and purposeful about modeling leadership and instruction that reflected collaboration and proven effective practices. Instructional strategies were modeled to support a community of learners among the participants, just as instructors hoped the participants would strive to build within their schools. The focus throughout the program was always on the mathematics. Course instructors brought attention to the mathematics content; the developmental progression of the mathematical concepts, how children make sense of the mathematics, and which pedagogical moves afford students opportunities to make sense of the mathematics and to become better mathematical thinkers. Small group and whole group discussions were grounded in classroom practice by incorporating written and video cases of mathematics lessons. Cooperative group work was focused on mathematics content and mathematics content pedagogy and writing assignments required participants to reflect about transferring ideas from class discussions and projects into their practice. Teachers in the courses and instructors functioned as colleagues, sharing knowledge gained from their diverse practices and experiences which modeled how teachers and a mathematics specialist work together.

Instructors modeled effective questioning techniques and used formative assessment to help participants construct their understanding of the mathematics. The on-going formative assessment informed the instructors about connections students were making among the courses in order deepen their understanding of the mathematics content in a way that would strengthen their ability to recognize and articulate how children construct mathematical understandings. The case studies of classroom lesson used during the content courses supported why mathematics specialists needed a flexible way of thinking about mathematics.

Supporting inquiry-based learning was modeled in all courses through projects, tasks, and class discussions. Carefully constructed assignments allowed participants to develop and communicate their mathematical and pedagogical understanding. Problems involved solving a mathematical task in a collaborative group or just turning and talking to a shoulder partner about using a different strategy. In leadership courses, participants conducted student interviews, observed in other teachers classrooms, coached one-on-one with a mathematics teacher, met with their principal, and examined school data to determine school needs and planned professional development. A culminating project in *Leadership III* was a lesson study project in which a team of teachers was responsible for writing, implementing and assessing an inquiry-based lesson. The lesson study experience required all the knowledge and skills developed in the leadership courses. It provided students with the opportunity to reflect on their strengths and weaknesses as a communicator and a collaborator, working with a team to impact student learning.

Lesson learned. It became clear early in the VMSI activities that participants needed to be challenged more with oral presentations and writing assignments, so as participants progressed through the program writing and presentation assignments were given both in the mathematics and the leadership courses. The instructors recognized that writing, as a reflection of one's analytical and communication skills is of critical importance to a mathematics specialist. As mathematics specialists improved their writing skills, instructors found that other communication skills were strengthened. Clear and well thought out oral communication is necessary when called upon to summarize research findings for their principal and staff, to share data reports, to share the school's vision for the mathematics program. In addition, specialist must interact with teachers in developing more engaging lessons, lead professional development for small and large groups that meet the needs of teachers with diverse experiences, work with grade level teams that may be reluctant to share, talk with parents that do not understand the school's curriculum.

In addition to modeling proven effective teaching practices, the instructors realized the need to make those practices explicit. Instructors "stepped in and out" of their role as facilitators to talk specifically about facilitation moves. This can generate a meaningful discussion about how the instructor designed the activity; or why they decided to ask one student to share their idea prior to another; or why they had students chart their ideas; or presented students with a particular focus question; or why the groups were changed for a particular activity. Being explicit about teaching moves, and coaching moves point out the complexity and magnitude of the decisions that mathematics specialist must make every day. The goal was that students come to realize the more carefully they plan and anticipate their teaching and coaching moves, they will become increasingly more effective in their role as coaches of teachers.

Time spent in developing professional networks and nurturing collaborations is time well spent. The collaboration that evolved and matured among the VMSC, the VDOE, Virginia's K-12 mathematics leaders, as well as, mathematicians and mathematics educators in institutes of higher education has led to a comprehensive program to prepare mathematics specialists. The research, planning, coordination and effective evaluation during the grant-funded projects have been important in refining the VMSPP and in adding to the knowledge base about mathematics specialists at the national level.

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Section 3:

Mathematics and Leadership Courses Annotated Syllabi

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Mathematics and Leadership Courses
Annotated Syllabi

What follows is a description, presented as sample annotated syllabus for each of the six mathematics content courses and three mathematics education leadership courses that make up the VMSPP. As described in Section 1 of this report, the courses were developed by VMSI with the support of a SCHEV grant and four NSF grants. The courses were designed to prepare successful classroom teachers for the Virginia K-8 Mathematics Specialist license endorsement and to assume the role of a mathematics specialist. While multiple universities collaborated in developing and piloting the courses each degree-granting university included the program requirements for that university along with the content of the VMSPP courses to make up a complete masters degree program. In addition to the core content and leadership courses described below, two additional courses were developed; *Mathematics for Diverse Learners* and *Research in Mathematics Education*. These courses are not included in this report since the specific content of similar courses differed at the various universities.

The annotated syllabus for each course that follows is presented in a similar format. A description of the course goals followed by a course overview, course format as well as a sample of key project assignments, and course materials; primary student texts, instructor primary resources, and some supplementary readings that have been used in the courses. A course outline includes a sampling of topics and essential questions, as well as a sample lesson plan or the guidelines and rubrics for a project. It should be noted that while the course goals remained constant, instructors determined the specific activities used in each class to support students in achieving the goals.

For the purposes of this report, the mathematics course outlines are based on a 2-week summer institute with classes meeting for ten days and up to eight hours each day or a semester course meeting fifteen times for three hours. The mathematics education leadership course outlines are based on eight days and six hours each day.

Numbers and Operations for Mathematics Specialists

Numbers and Operations is a 3-credit hour graduate mathematics course designed to prepare teachers with at least three years of classroom teaching experience to become school-based mathematics specialists. This is the first course in the program for future K-8 mathematics specialists. The course focuses on the number and operations standard described in the National Council of Teachers of Mathematics (NCTM) *Principals and Standards for School Mathematics* and the number and number sense and computation strands described in the *Virginia's Standards of Learning for Mathematics* and the *Curriculum Frameworks* (VDOE). The course develops a comprehensive understanding of the base ten number system, its structure, the role this structure plays in problem solving and computations, and the properties of arithmetic that form the foundation for algebra. Attention is given to connecting these mathematics concepts to school students' thinking as they solve problems and construct their understanding of the number system and develop their proficiency in arithmetic computation.

Course Goals

The goal of the *Number and Operations* course is to engage students in constructing a deeper conceptual understanding of the base ten number system, to identify the relationships among the four operations, and to understand the mathematics that underpins different computational strategies for whole numbers and decimals. In addition, students develop an understanding of fractions as numbers and how that understanding supports comparing, ordering, and operating on positive fractions. Students also explore the role that multiple representations play in developing mathematical understanding and presenting mathematical arguments. This course will:

1. Develop an understanding of the structure of the base ten number system that influences learning to count, performing operations with multi-digit numbers, and working with decimal numbers.
2. Develop the knowledge and skill to represent and interpret quantitative situations verbally, pictorially, and symbolically.
3. Investigate a variety of situations modeled by the four basic arithmetic operations; addition, subtraction, multiplication, and division, and examine various representations of the four operations.
4. Develop the knowledge and skill to recognize generalizations in different computational situations that lead to generalizations for understanding and representing the properties that support the operations verbally and symbolically.
5. Develop an understanding of positive fractions as numbers and how an understanding of the principals that govern whole number operations need to be expanded to operate with positive fractions.
6. Study the mathematical progression of concepts and how children develop their understanding of number and operations using case studies.
7. Investigate mathematics ideas about number and operations by engaging in problem solving, reasoning, conjecture, and developing and defending mathematical arguments.

Course Overview

The structure and logic of the base ten number system are fundamental to understanding and reasoning about the four arithmetic operations and developing computational fluency. Often elementary and middle school mathematics teachers have a limited understanding of the structure of the Hindu-Arabic numeral system. That is an understanding of the supporting structure and logic of a positional base ten number system. In addition, teachers need to extend their understanding of quantity represented by whole and decimal numbers and develop their conceptual understanding of quantity represented by rational numbers. During the course, teachers make connections among the four operations for whole numbers and positive rational numbers. In summary, the purpose of this course is to provide opportunities for prospective mathematics specialists to build their knowledge, skills, and mathematical dispositions to support teacher and student learning in number and operations.

The core of the instructional program is developed around two of the Developing Mathematical Ideas (DMI) modules which include rich mathematical tasks and case studies and then supplemented with additional readings, mathematics activities, and analysis of school classroom artifacts. The course begins with DMI *Building a System of Ten (BST)* as the primary text to explore the base ten number system structure, consider how that structure supports multidigit computational procedures and examine how basic concepts of whole numbers reappear when the set of numbers is expanded to include working with decimals. The second half of the course moves to the DMI module, *Making Meaning of Operations* (MMO) where participants examine the actions and situations modeled by addition, subtraction, multiplication, and division in order to make meaning of each of the operations and the relationships among the operations. Using the print and video cases in MMO students first, examine young children's counting strategies as they encounter word problems and then move to an examination of the four basic operations on whole numbers. Students then revisit the operations in the context of positive fractions.

Throughout the course, participants engage in making conjectures, developing generalizations, and making mathematical arguments. Through classroom discussions, students build a working knowledge of the properties of arithmetic which are formalized in the Functions and Algebra course for mathematics specialist. Decimal numbers are given a thorough treatment in the *Numbers and Operations* course. Participants spend some time in *Numbers and Operations* making sense out of fractions as numbers and the role that fractions play in quantifying different situations. The principles governing computation with whole numbers are re-examined in light of the four basic operations with positive fractions. The *Functions and Algebra* I course provides an opportunity for students to expand their understanding of real numbers to include integers and computation with integers. In the *Rational Number and Proportional Reasoning* course students engage in a more thorough study of fractions.

Instructors should be aware that the ideas introduced in this course will be examined again through an algebra lens in the *Functions Algebra* course. For example, as participants in that course look at generalizations and then laws of arithmetic, they will revisit where notions of these laws began to develop in the *Numbers and Operations* course. For instance, the Distributive Property is used by students when they create array models for multiplication, with both whole numbers and rational numbers: do participants recognize 6 ½ x 4 = 6 x 4 + ½ x 4 as the distributive property? In other words, do they see it as more than the property, but as a strategy to help with computation?

Course Format and Key Activities

A variety of formats has been employed to teach The *Numbers and Operations* course. For example, it has been taught as two-week residential and commuter summer institutes with 54 hours of class time and significant in-class work and homework assignments and as a school-year course with 15 three-hour sessions in one semester or spread over two semesters. The timeframe in which the course is offered can impact participants experience and the instructors will need to plan accordingly. One benefit of the summer institutes is that students immerse themselves in the course and have the opportunity for additional collaboration with their peers and the instructors after class hours. This is particularly true for the residential institutes. However, in the summer, there are few if any relevant opportunities readily available to immediately do the mathematics with their students, to interview students about their understanding of the mathematics, or write cases based on their students. Instructors in the summer institutes use video of student interviews and bring samples of student work from other sources to provide opportunity for students to experience analyzing where students are in the developmental progression for number and make recommendations for next instructional steps

The instructional methodology includes small group and whole group discussions anchored in written and video cases of students mathematics thinking; cooperative group work around mathematics content and mathematics content pedagogy; and analyzing student interviews, student work, and cases from participants' practice. While developing the participant's mathematics content knowledge for teaching is the focus of the course it is just as important to include the case studies as a venue to deepen their understanding of how children make sense of the mathematics. The case studies also bring validity to teaching mathematics for conceptual understanding as well as for computational fluency. Class discussions about the mathematics as well as the cases become more robust as participants develop deeper understandings of the mathematics and the developmental progression of various mathematics topics. Course instructors bring explicit attention to the mathematics content, the developmental progression of the mathematics concepts, how children make sense of the mathematics, and which pedagogical moves afford students opportunities to become mathematical thinkers. Instructors intentionally model inquiry teaching throughout the course

Ongoing informal and formal formative assessment is an important component of the course. Instructors continually adapt the class activities and course projects to support participants to construct understanding and make connections to their classroom practice that deepen their understanding of how school students make sense of the mathematics. The course projects include maintaining a portfolio of the mathematics problem sets assigned for homework, completing two student interviews, analyzing two sets of student work, and maintaining a reflection journal throughout the course. Rubrics are provided for each project and writing is a component of each project. In addition, to the ongoing informal and formal formative assessments, there are three summative assessments, a midterm test, a cumulative final exam, and a final reflection synthesis paper.

Course Materials

Listed below are the primary student and instructor texts for the course. In addition, instructors include supplementary readings from sources such as NCTM journal articles.

The Primary Student Texts

Schifter, D., Bastable, V., & Russell, S.J. (2010). *Developing mathematical ideas, Number and operations, part 1: Building a system of tens, Calculating with whole numbers and decimals. Casebook*. Upper Saddle River, NJ: Pearson.

Schifter, D., Bastable, V., & Russell, S.J. (2010). *Developing mathematical ideas, Number and operations, part 2*, Making *Meaning for Operations in the domains of whole numbers and fractions. Casebook.* Upper Saddle River, NJ: Pearson.

Instructor Primary Resources

Schifter, D., Bastable, V., & Russell, S.J. (2010). *Developing mathematical ideas, Number and operations, part 1: Building a system of tens, Calculating with whole numbers and decimal. Facilitator's guide and video*. Upper Saddle River, NJ: Pearson.

Schifter, D., Bastable, V., & Russell, S.J. (2010). *Developing mathematical ideas, Number and operations, part 2*, Making *Meaning for Operations in the domains of whole numbers and fractions. Facilitator's guide and video*. Upper Saddle River, NJ: Pearson.

Instructors can retrieve additional information from the developers and various implementers of the DMI materials at http://www2.edc.org/cdt/dmi/dmiless.html.

Instructor Supplementary Resources

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

Cramer, K., Behr, M., Post, T., & Lesh, R. (1997). *Rational number project fraction lessons for the middle grades level 1*. Retrieved http://www.education.umn.edu/rationalnumberproject/97_2/97_2.html

Cramer, K., Behr, M., Post, T., & Lesh, R. (1997). *Rational number project fraction lessons for the middle grades level 2*. Retrieved http://education.umn.edu/rationalnumberproject/rnp2.html

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Fosnot, C. T., & Dolk, M. (2001b). *Young mathematicians at work: Constructing multiplication and division*. Portsmouth, NH: Heinemann.

Fosnot, C. T., & Dolk, M. (2001c). *Young mathematicians at work: Constructing fractions, decimals, and percents.* Portsmouth, NH: Heinemann.

Kamii,C. (2000) Young children invent arithmetic: Implications of Piaget. New York, N.Y., Teachers College Press.

Kilpatrick, J., Swafford, J., and Findell, B. (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academy Press.

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Richardson, K. (2012). *How children learn number concepts: A guide to the critical learning phases.* Bellingham, WA: Math Perspectives Teacher Development Center.

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Supplementary Readings for Students

Behrend, J. (2003). Learning disabled students make sense of mathematics. *Teaching Children Mathematics*, *9(*5), 269-273.

Clark, D., Roche, A., Mitchell, A. (2008). Ten Practical Tips for Making Fractions Come Alive and Make Sense. *Mathematics Teaching in the Middle Sch*ool, *13(*7), 372-379.

Kaplan, R.G., King, B, Dickens, N., Stanley, V. (2000). Teacher-clinicians encourage children to think as mathematicians. *Teaching Children Mathematics*, *6(*6), 406-411.

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Pickreign, J. & Rogers, R. (2006). Do you understand your algorithms? *Mathematics Teaching in the Middle Sch*ool, *12(*1), 42-47.

Riddle, M. (2000). Fractions: What happens between kindergarten and the army? *Teaching Children Mathematics, 7(*4), 20-206.

Russell, S.J. (2000). Developing computational fluency with whole numbers. *Teaching Children Mathematics.* 154-158.

Torrence, E. (2003). Learning to think: An American third grader discovers mathematics in Holland. *Teaching Children Mathematics, 10(*2), 90-93.

Wilson, P. (2001). Zero: A special case. *Mathematics Teaching in the Middle School. 6(*5), 300-303.

Numbers and Operations Course Outline: Topics and Essential Questions

The overview displayed in Figure 3 presents the scope and sequence of topics in the *Numbers and Operations* course taught in 15 weekly 3-hour class sessions. The overview identifies the topics and essential questions for each class and resources used to support each class. The *Numbers and Operations* course relies heavily on the mathematics activities and the print and video case studies included in the two Developing Mathematical Ideas (DMI) modules. Instructors supplement and extend the mathematics through additional mathematics activities and readings. The course textbooks are noted in the outline as follows.

- **BST** indicates the *Developing mathematical ideas, Number and operations, part 1: Building a system of tens, Calculating with whole numbers and decimals. Casebook.*

- **MMO** indicates the *Developing mathematical ideas, Number and operations, part 2*, Making *Meaning for Operations in the domains of whole numbers and fractions. Casebook*.

Figure 3. Numbers and Operations Course: Overview of Topics.

Class	Topics/Resources	Essential Questions
1	Operation of addition Traditional and alternative addition strategies Number talks and mental math as an instructional tool Commutative and associative property of addition BST Chp 1	How do the base-ten structure of the number system and the properties of the operations shape the strategies for multidigit computation? In what ways can numbers be composed and decomposed? How do different visual or physical representations of number highlight the tens structure of the number system?
2	Base ten structure Magnitude Powers of 10 Place value Traditional and alternative subtraction strategies History of the Hindu-Arabic	What is the structure of the base-ten or Hindu-Arabic numeration system? What are the keys to understanding any numeration system? What is the role of unitizing in developing an understanding of place value? What happens when a number is multiplied by a power of

	numeration system Activity: Xmania http://mathinscience.info/teach/612_math/math68/count_on_it/xmania_backup/xmania.htm BST Chp 2	10 such as 10, 100, 1000 and so forth?
3	Operation of subtraction Models for subtraction Generalizations: a+b = (a+c) + (b-c) a-b = (a+c) - (b+c) a+0=a a-a=0 BST Chp 3 Sample lesson for class 3 follows the course overview.	How do computational strategies for multidigit addition and subtraction rely on the place value and the structure of the base-ten number system? How do different representations illustrate quantity and how are different representations related to each other? How do different representations illustrate the operations of addition and subtraction and how are different representations related to each other? What is the role of zero and why is understanding zero challenging for students? What does it mean to be fluent with computation?
4	Operation of multiplication Traditional and alternative strategies for multiplication Models for multiplication Commutative and associative property of multiplication Distributive property BST Chp 4	What is the relationship between multiplication and addition? How do the procedures for calculating a multidigit multiplication problem rely on place value and the structure of the base-ten number system? How do different representations illustrate multiplication and how are different representations related to each other? What role does the distributive property play in multiplication?
5	Operation of division Traditional and alternative strategies for division Partitive division Quotative division BST Chp 5	How do the procedures for calculating a multidigit division problem rely on the base-ten structure of the number system? How do the strategies for decomposing numbers work or not work for division? What models for representing and thinking about partitive and quotative division support student understanding of division?

		How can using and analyzing different representations support students' understanding of multiplication and division? How do different contextual situations lead to different models for division?
6	Place value representations of numbers less than1 Comparing and ordering decimal numbers Addition of decimal numbers Subtraction of decimal numbers BST Chp 6	How do students use what they understand about whole numbers and place value when they begin working with decimals? What new ideas do students need in order to understand decimals as numbers and the role they play in representing quantity? How do different representations illustrate decimals and how are different representations related to each other? Why do the same principles that govern whole number addition and subtraction apply to addition and subtraction of numbers involving decimals?

Class	Topics/Resources	Essential Questions
7	Multiplication with decimals Division of decimals BST Chp 7 Take-home Midterm Test	How do the same principles that govern whole number multiplication and division apply to multiplying and dividing numbers involving decimals?
8	Addition and subtraction as inverse operations If $a + b = c$ then $a = c - b$ and $b = c - a$ Identity element for addition Number line as a tool to represent the different models for subtraction Classification of addition and subtraction word problems MMO Chp 1	What do children understand when they use counting strategies to solve problems before they learn to add and subtract? Why can the same situation be represented by addition and subtraction sentence? How can the inverse relationship between addition and subtraction be developed using the number line? How do different contextual situations lead to different models or interpretations for subtraction? How can different representations support students' understanding of addition and subtraction?
9	Multiplicative reasoning Additive reasoning Multiplication and division as inverse operations If $a \times b = c$ then $a = c \div b$ and $b = c \div a$ Identity element for multiplication 0 as a factor, as a divisor, and as a dividend MMO Chp 2	What does it mean to understand the *concept* of addition, subtraction, multiplication, and division? How can examining different number sentences that model a single situation develop an understanding of the relationships between operations? What does it mean to reason additively? multiplicatively? How do different contextual situations lead to different models for division? How does dividing a whole into parts lead to thinking about fractions as numbers? What role does context play when interpreting the remainder in division? What relationships exist among the four basic arithmetic operations?

10	Rational numbers Partitioning Iterating Unit fraction Density of fractions Area, set, and measurement model for representing fractions Fractions as numbers representing different relationships MMO Chp 3	How do the area, set, and measurement models for fractions highlight different ways of thinking about fractions as numbers? How do different contexts support the five main interpretations: fractions as parts of wholes; fractions as the result of dividing two numbers; fractions as the ratio of two quantities; fractions as operators; and fractions as measures? What is the role of the numerator and the denominator of a fraction? How can partitioning a whole and iterating to create a whole support understanding fractions as numbers?
12	Unit fractions Equivalent fractions Comparing and ordering fractions MMO Chp 4 The operations of addition and subtraction with fractions Common units MMO Chp 5	Why is knowing the unit or whole when working with fractions necessary? What does it mean to have a unitary view of fractions? What does it mean when two fractions are equivalent? Why can the same point on the number line be named by fractions with different names? Why can the same area of a region be named by fractions with different names? What reasoning strategies support students in efficiently comparing and ordering fractions? Why does changing the unit result in different fractional names for the same quantity? What does it mean to have "fraction sense"?
13	The operations of multiplication and division with fractions Inverse property of multiplication MMO Chp 6	How does the meaning of multiplication and division need to be extended when the set of numbers operated on is expanded to include both whole numbers and positive fractions? How do different verbal, visual, and physical representations of fractions highlight the different computation strategies for multiplication and division with fractions? What concepts support understanding the invert and multiply algorithm for division of fractions? What concepts support understanding the common denominator algorithm for division of fractions?
14	Division of fractions MMO Chp 7 Review for the final exam MMO Chp 8	What role does the unit play in making sense of the remainder of division with fractions? How are the partitive and quotative types of division seen in problems with fractions? What does it mean to model a situation with an arithmetic

		sentence or expression? What are the four categories that support context that involve multiplication and division with positive fractions? (MMO FG p. 237) How are common fractions, decimals and percents alike and different?
15	Final Exam	

Sample Lesson Plan for Class 3, Numbers and Operations Course

Textbook: BST Chapter 3 Making Sense of Addition and Subtraction Algorithms
Materials: base-ten blocks, interlocking cubes, HO BST Chp 3 Focus Questions p. 115, HO BST FG Math Activity p. 114, HO with guidelines for the Student Work Samples Writing Assignment.

Essential Questions
How do computational strategies for multidigit addition and subtraction rely on the place value and the structure of the base-ten number system?
How do different representations illustrate quantity and how are different representations related to each other?
How do different representations illustrate the operations of addition and subtraction and how are different representations related to each other?
What is the role of zero and why is understanding zero challenging for students?
What does it mean to be fluent with computation?

Math Talk Warm Up (20 minutes)
Present the expressions, 128 + 85 and 63 - 25, one at a time and spend about 10 minutes per problem using the steps below to process each problem.

- Individually: Find two mathematically different ways to solve each problem mentally.
- Table groups: Discuss the mathematics behind the different methods used in the table group. Instructor circulates and picks out several different methods to share the whole group.
- Whole group: Pick several participants to share their one of their methods. Then ask for a whole group discussion about how the different ways are mathematically different.

Link Back to Class 2 and Xmania (20 minutes)
Have a handout or write the following on the board and allow 10 minutes for students to work and 10 minutes to share.

- When working in base 4 what numerical symbols or digits would be used to represent the quantity?
- Draw the base 4 blocks or pieces to represent the base-ten number 29.

- In base 4 what place value is represented by each position or place in the number ___ ___ ___ ___

Math Activity Addition and Subtraction Strategies (45 minutes) Whole group introduction: Need handout Math Activity: Addition and Subtraction Strategies from BST FG Chp 3 p. 114.Complete question #1 together. Prepare participants to develop the poster for 1d by asking them to analyze and describe each student's work displayed in 1a, 1 b, and 1 c. The descriptions should focus on the mathematics and properties that support the work. After the discussion work a class to develop a poster for 1d which will include a verbal description of the problem, a pictorial representation that models the strategy, and a story context that matches the actions in the story. When the class poster is complete, discuss the mathematics and any generalization that may lead to a property, the questions below can be used if needed.

1) What does this work tell about addition?
2) How do we maintain equivalence if we change the addends or the problem? What do we learn about equivalence and addition from this work?
3) If participants are only able to describe the generalization in words, lead them to the symbolic representation, $(a + b) = (a - c) + (b - c)$.

- Small group: Assign each table group one of the student subtraction strategies in #2 to analyze. They should make posters similar to the one made in the whole group for #1 regarding addition.
- Whole group: Each will display his or her poster for a gallery walk.

Posters Gallery Walk and Math Discussion (20 minutes)

- While viewing posters, consider the following questions:
  1) What mathematics is evident in these procedures?
  2) What do students need to understand in order to compute in these ways?
- Whole Group Math Discussion, addition and subtraction strategies. First, make sure 2 b,c, and e are clear.
  1) How is 2b the same or different from 1d?
  2) What do you notice when you examine 2c?
  3) What comments do you have about 2e?
  4) What generalizations did you note for any of the subtraction strategies?
  5) What similarities and differences did you note between the addition and subtraction strategies?

DVD for BST Chp 3 (10 minutes)
If time is short, use the DVD at the beginning of class 4 as a link back to class 3.

- The DVD will be watched straight through; any points the participants want to discuss should be written down and brought forward in case discussion. Ask them to think about the following questions as they watch the DVD and make notes of specific evidence from what the children are doing and/or saying.

1) How is an understanding of place value necessary for an understanding of addition and subtraction?
2) How does a realization of a generalization that leads to understanding a property develop as children's addition and subtraction strategies develop?

BST Chapter 3 Case Discussion (30 minutes)
Many of the ideas addressed in the focus questions are brought out in the Warm Up and the Mathematics Activity so most of the time can be used to bring forth summary ideas.

- Small group discussion will be guided by the Focus Questions for BST CHP 3.
- Whole group discussion is guided by the following questions.
    1) How are the mathematics strategies we see students create based on the same underlying principles as the standard algorithms?
    2) What are those principles?
    3) How does the development of these understandings serve as the early development of algebraic thinking and the understanding of algebraic properties?

Reading and Discussion, What does it mean to be computational fluent? (20 minutes)

- Read the short article, Russell, S.J. (2000). Developing computational fluency with whole numbers. *Teaching Children Mathematics.* Then, write a "matchbook" definition computational fluency that you can use when talking with teachers and parents. If there is not sufficient time in class, this can be assigned for homework.
- Small Groups will share their definition and pick one to share with the whole class.
- Whole Group: Have small groups share and then pose the question, as you compare each definition what was the key idea(s) that emerged.

Exit Card Prompt:
How do you think the standard algorithm best fits the curriculum and/or instruction?

Homework for Class 5

1. Writing Assignment: Analyzing subtraction algorithms. Provide HO BST FG p. 116. This will be used in the next class for discussion and then will be put into the portfolio which is due in Class 5.

2. Writing Assignment: Students' Work Samples. We will share in groups next week, and you will turn in your paper with student samples at the end of class.

Writing Assignment Guidelines
In this course, we will explore the ways students engage with the topics of the elementary and middle school mathematics curriculum. Part of our next session will be devoted to discussion of the mathematical goals we have for our students. In preparation for this discussion, please complete the following assignment.

Ask your students a question (give them a problem) relating to multidigit computation. The problem can involve addition or subtraction depending on the grade level you teach.

If you teach younger children, ask a question focused on making sense of the numbers between 10 and 20. In order to get the most out of your students, it is a good idea to give a problem that has a context—this way if the student is not sure what to do, there may be some point of entry.

For instance, a young child might be asked to work with the problem:

> There were nine eggs in a basket. The farmer collected 6 more eggs and put them in the basket. Now how many eggs are in the basket?

A 4th-grade student might be asked:

> The Murrays were driving 143 miles to Washington, DC. After 87 miles, they took a break, How many miles did they still need to drive?

A 6th-grade student might solve:

> How many packs of gum do I have if gum comes 14 sticks to a pack and I have 168 pieces of gum?

Examine the work you get from your students. Choose three students to write about: one whose mathematical work shows a good understanding of the mathematics, and two whose work show some misconceptions about the mathematics. Then write your analysis of these three students' work. For each student, your analysis should include:

a) What does the student understand? What is the evidence?
b) What is the student missing that would enable more sophisticated mathematical work?
c) Based on what your analysis, what is a learning goal for the student?
d) What instructional strategies would support the student's learning goal?

Bring copies of your students' work to the class. Be sure to remove or mark out the student's name. Label the papers Student A, Student B, and Student C.

Rational Numbers and Proportional Reasoning for Mathematics Specialist

Rational Numbers and Proportional Reasoning for Mathematics Specialists is a 3-credit hour graduate mathematics course designed to contribute to the preparation teachers with at least 3 years of classroom teaching experience receive to become school-based mathematics specialist. The course will develop a deep understanding for teaching rational numbers, proportionality, and instructional strategies to facilitate the instruction of this content in the K-8 curriculum: interpretation and operation with fractions, decimals, percents, ratios and proportions. Through class discussions and class activities participants will connect these rational number and proportional reasoning concepts to related concepts that support elementary and middle school concepts such as dilations, scale factor, similar figures, probabilistic concepts, statistics, slope as a constant rate of change, and functions. Attention will be given to interpreting and assessing students' thinking.

Course Goals

Demonstrate depth and sophistication of knowledge through the ability to complete mathematical problems, discuss and explain the mathematical concepts connecting rational number and proportional reasoning ideas to K-8 arithmetic topics. This course will:

1. Use skills and knowledge of rational numbers to support the development of proportional reasoning:

- Connect understanding of fair sharing with fractions and rational numbers.
- Identify the variety of concepts fractions represent: part whole, measurement, quotient/division, ratio, and operator.
- Investigate different fraction models: area, length or measurement, and set models.
- Make meaning of equivalent fractions and the contexts and models for the different names for fractions.
- Develop an understanding of fraction computations using models and connect these models to the efficient algorithms.
- Explore connections between fractions, decimals, and percents.
- Consider the members of the family of rational numbers and their relationship to the Real Number System.
- Investigate the relationship between ratios, rates, unit rates, and proportions.
- Identify the differences between additive and multiplicative thinking.
- Investigate the role rational numbers play in dilations, scale factor, similar figures, probabilistic concepts, statistics, and functions.

2. Develop an understanding of how children develop rational number understanding and proportional reasoning using case studies and analyzing student work**.**

3. Develop mathematical habits of mind to support the work of a mathematics specialist.

Course Overview

Historically students have had profound difficulties learning and applying rational number and proportional reasoning concepts. One particularly challenging area is understanding and operating with fractions. Fractions are considered by many to be among the most problematic topics in the elementary school curriculum. However, understanding fractions is only the beginning of the journey toward rational number understanding. During the middle school years, students are provided opportunities and experiences to apply and extend their knowledge of rational numbers as they develop proportional reasoning. It is, however, difficult for many and a lack of understanding can become an obstacle to further progress in mathematics and other subjects dependent on mathematics.

In the first half of the class, participants use the books *Young Mathematicians at Work* (YMW) with DVDs and *Teaching Fractions and Ratios for Understanding: Essential Content Knowledge and Instructional Strategies for Teachers* (Lamon). In addition, instructors use many activities from *Making Sense of Fraction Ratios and Proportions: 2002 NCTM Yearbook* and *Elementary and Middle School Mathematics: Teaching* Developmentally (Van de Walle). A number of supplementary materials have been used in the class and are mentioned in the course outline and cited in the resources for instructors and students. The mathematics activities selected enhance the study of rational numbers and the development of proportional reasoning. Specifically, they help develop the idea of a unit, or whole, and connect it with fractional parts. They introduce the various interpretations of fractions and the models used to represent them. Through these experiences, attention is drawn to the number of ways in which rational number equivalence can be expressed. Participants then experience computation with rational numbers, which extends ideas encountered in the Numbers and Operations course while visiting many new ideas and processes. The importance of modeling rational number operations and connecting these models to efficient algorithms is emphasized.

In the second half of the course participants use the book *Improving Instruction in Rational Numbers and Proportional Reasoning: Using Cases to Transform Mathematics Teaching and Learning* (Using Cases), many activities from *Making Sense of Fraction Ratios and Proportions: 2002 NCTM Yearbook* and a variety of other sources mentioned in the course outline and cited in list of resources for instructors and students the. Proportional reasoning will be developed through activities involving comparing and determining the equivalence of ratios and solving proportions in a variety of problem-based activities. Connections between equivalent ratios and equivalent fractions will be made and highlighted by attention to the multiplicative process. Participants will preview connections between the *Rational Numbers and Proportional Reasoning* course and future courses as they investigate the role rational numbers play in a variety of content areas. Participants will investigate similar figures as a visual representation of proportions and ratios. Ratio tables will be examined as a way to organize and quantify variability between two different amounts. The existence of equivalent ratios will help participants make statistical conclusions, determine the outcome events, and graph straight lines. Instructors should draw attention to the importance of developing proportional reasoning and its importance to the deep understanding of many topics found in elementary and middle school mathematics curriculum.

Course Format and Key Activities
The course has been taught in a variety of formats. Those most frequently chosen, the two-week residential summer institutes and semester courses result in a variety of impacts on participants, In the two-week institutes, there is typically 54 hours of class time and significant daily in-class work and homework assignments. In a semester course, there are 15 three-hour sessions. In the summer, students immerse themselves in the work and have the opportunity for additional collaboration with their peers after class hours. However, in the summer, there are few if any relevant opportunities to transfer the coursework to their own classrooms. In semester courses, it much more likely that teachers will be able to implement strategies they learn in classroom situations.

The instructional methodology assumes a student-centered, inquiry model and makes use of small groups and whole group discussions based on mathematics tasks, cases, and cooperative group work around mathematics content and mathematics content pedagogy. Doing mathematics together is an important part of the course. Mathematics task and activities are included in the course text and instructors supplement with additional tasks based on students' needs. In addition, when the course format allows, the students transfer their learning from the course to their own classroom practice by analyzing their own students' work samples, interviewing individual students, and writing cases from their classroom practice. Course instructors bring attention to the mathematics content, the developmental trajectory of the mathematical concepts, to mathematical sense making in children, and to pedagogical moves that afford students opportunities to become mathematical thinkers.

While mathematics is the focus of the course, it is also important to include the textbook case studies as part of the participant's experience as a context to deepen students' understanding of how K-8 learners make sense of the mathematics. The case studies offer validity to the learner-centered manner of teaching and making sense of mathematics. In addition, the cases illustrate the critical role the teacher plays in selecting tasks and orchestrating classroom discussions. Students, with instructor guidance, engage in robust conversations about the mathematics in the cases and, as a result, attain a deeper understanding of the mathematics. There are occasions when the instructor needs explicitly to connect the case study discussion with the mathematics discussions or to supplement with activities that encourage additional amplification of the mathematics.

Instructors and students engage in ongoing formative assessment, and as students construct their own understanding of the mathematics, and make connections throughout the course that deepen their understanding of the mathematics content. Specific writing prompts are provided by the instructor throughout the course of the evaluation and evidence of evolving understanding. Two summative assessments are administered, a midterm assessment and a cumulative final exam.

Course Materials
The Primary Student Texts

Fostnot, C. & Dolk, M. (2002). *Young mathematicians at work: Constructing fractions, decimals, and percents*. Portsmouth, NJ: Heinemann. (and DVD)

Lamon, S. (2005). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers (2nd ed.)*. Mahwah, NJ: Lawrence Erlbaum Associates.

Litwiller, B. & Bright, G. (2002). *Making sense of fractions, ratios, and proportions*. 2002 Yearbook. National Council of Teachers of Mathematics, Reston, VA. (and Classroom Activities)

Smith, M., Silver, E., & Stein, M. (2005). *Improving instruction in rational numbers and proportional reasoning: Using cases to transform mathematics teaching and learning*. New York. NY: Teacher's College Press.

Instructor Primary Resource:

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM. ***(A student text in Leadership I)***

Van de Walle, J., Karp, K., & Bay-Williams, J.M. (2010). (7th edition). *Elementary and middle school mathematics: Teaching developmentally*. New York, NY: Pearson Education (Allyn & Bacon). ***(A student text in Leadership I)***

Activities/Readings Recommended to Instructors:

AIMS Educational Foundation. (2008). *Statistics and probability*. Fresno, CA: AIMS Publishing.

Cameron, A., Jacob, B., Fosnot, C., & Hersch, S. (2006). *Working with the ratio table, grades 5-8, mathematical models: Facilitator's Guide.* Portsmouth, NJ: Heinemann (with DVD).

Cameron, A., Werner, S., Fosnot, C. & Hersch, S. (2006). *Minilessons for operations with fractions, decimals, and percents, grades 5-6: Facilitator's Guide*. Portsmouth, NJ: Heinemann (with DVD).

Cramer, K, Wyber, T. & Leavitt. S. (2008). The role of representation in fraction addition and subtraction. *Mathematics Teaching in the Middle School,* 13*(*8), *490-496.*

Empson, S. & Levi, L. (2011). *Extending children's mathematics: Fractions and decimals*. Portsmouth, NJ: Heinemann

Erikson, S., Anderson, D., Hillen, J. & Wiebe, A. (2000). *Proportional reasoning*. B. Cordel (Ed.). Fresno, CA: AIMS Publishing.

Flea-sized surgeons a middle-school mathematics unit focusing on surface area, volume, and scale. (1994). California: Lawrence Hall of Science, University of California at Berkeley

Goldstein, D. & Jackson, B. (1994). *Fractions, decimals, ratios, & percents: Hard to teach, hard to learn*. C. Barnett (Ed.). Portsmouth, NJ: Heinemann.

Hargrove, T. (2015). *Making and investigating fraction strips*. Retrieved from: http://illuminations.nctm.org/Lesson.aspx?id=1724

Hersch, S., Tarlow, L., Fosnot, C. & Cameron, A. (2006). *Exploring playgrounds, a context for multiplication of fractions: Facilitator's guide.* Portsmouth, NJ: Heinemann (with DVD).

Lappan, G. (2009). *Bits and pieces I: Understanding fractions, decimals, and percents*. Boston, Massachusetts: Pearson.

Lappan, G. (2009). *Bits and pieces II: Using fraction operations.* Boston, Massachusetts: Pearson.

Lappan, G. (2009). *Bits and pieces III: Computing with decimals and percents.* Boston, Massachusetts: Pearson.

National Council of Teachers of Mathematics. (2015). *Fraction game: Fraction tracks*. Retrieved from: http://illuminations.nctm.org/activity.aspx?id=4148

Wiebe, A. (2005). *Looking at lines: Interesting objects and linear functions*. Fresno, CA: AIMS Publishing.

Supplementary Readings:

Bright, G., Joyner, J. & Wallis, C. (2003). Assessing proportional thinking. *Mathematics Teaching in the Middle School, 9(*3), *166-172.*

Empson, S. (2001). Equal sharing and the roots of fraction equivalence. *Teaching Children Mathematics, 7(7), 421-425.*

Langrall, C. & Swafford, J. (2000). Three balloons for two Dollars: Developing proportional reasoning. *Mathematics Teaching in the Middle School, 6(*4), 254-261.

Ortiz, E. (2006). The roll out fractions game: Comparing fractions. *Teaching Children Mathematics, 13(1), 56-62.*

Petit, M., Laird, R., & Marsden, E. (2010). *A focus on fractions: Bringing research to the classroom*. New York, NY: Routledge.

Seibert, D. & Gaskin, D. (2006). Creating, naming, and justifying fractions. *Teaching Children Mathematics, 7(7), 421-425*

Course Outline: Topics and Essential Questions

The overview displayed in Figure 4 presents the scope and sequence of topics in the *Rational Numbers and Proportional Reasoning* course taught in 15 weekly 3-hour class sessions. The overview identifies the topics and essential questions for each class and resources used to support each class. The course textbooks are noted in the outline as follows.

YMW: *Young mathematicians at work: Constructing fractions, decimals, and percents*.
Lamon: *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers (2nd ed.).*
Van de Walle: *Elementary and middle school mathematics: Teaching developmentally*
NCTM Yearbook: *Making sense of fractions, ratios, and proportions* (2002 Yearbook)
Using Cases: *Improving instruction in rational numbers and proportional reasoning: Using cases to transform mathematics teaching and learning*

Figure 4. Rational Numbers and Proportional Reasoning Course: Overview of Topics

Class	Topics/Resources	Essential Questions
1	**INTRODUCING FRACTIONS** Submarine Problem Hugh's Invention **YMW** Chapter 1 **Fractions, Decimal, Ratios, & Percents** (Goldstein)	What is the prerequisite knowledge that students must bring with them to understand fractions? What are fractions? How do we help students see fractional parts of a whole? How do we introduce fair sharing and partitioning a whole into equal-sized portions? What strategies can be used to subdivide "left-over" portions to fair share them? How do we build on what students intuitively know about fractions and the informal strategies they use to problem solve?
2	**FRACTION CONSTRUCTS: CONCEPTS FRACTIONS REPRESENT** Part-Whole Measurement Quotient/Division Operator Ratio **YMW** Chapter 4 **Lamon** Chapter 2 (Meaning of ¾?)	How can models be used to help students see that a fraction represents the *relationship* that exists between the part and the whole, not two separate numbers? What experiences help students see fractions as the concepts they represent (i.e. a fraction as an operator)? What experiences help students make sense of the different concepts fractions can represent? How do we illustrate why a representation is important and the kinds of contexts in which each representation is useful?

<table>
<tr><td></td><td>Van de Walle - activities</td><td></td></tr>
<tr><td>3</td><td>FRACTION MODELS: STRENGTHS AND LIMITATIONS
Area
Length
Set

YMW Chapter 5
NCTM YB Activities
Van de Walle – activities</td><td>How do I identify and record the fraction of a whole or group or measure?
How are the part and the whole identified in each model?
How do I explain how changing the size of the whole effects the size or amount of a fraction?
What experiences help students move fluently among the different types of models?
What experiences help them choose the most efficient model given circumstances?</td></tr>
<tr><td>4</td><td>FRACTION EQUIVALENCY AND COMPARISON
Equivalent Fractions
Comparing Fractions
Ordering Fractions

YMW Chapter 3
Lamon Chapter 10
NCTM website: Illuminations - Fraction
Strips (Hargrove)
NCTM YB Activities
Bits and Pieces I (Lappan)</td><td>How do I use concrete materials and drawings to understand and show understanding of fractions?
How do I explain the meaning of a fraction and its numerator and denominator, and use my understanding to represent and compare fractions?
How do I help students develop the idea of the density of fractions?
What activities help me develop the idea of betweenness?
What strategies can be used to solve estimation problems with fractions?</td></tr>
<tr><td>5</td><td>FRACTION ADDITION AND SUBTRACTION
Adding and Subtracting Fractions with Like Denominators
Adding and Subtracting Fractions with Unlike Denominators
Moving from Models to Algorithms

YMW Chapter 7 / Minilesson DVD for Operations with Fractions, Decimals, and Percents (Cameron)
NCTM website: Illuminations</td><td>How can models be used to compute fractions with like and unlike denominators?
How are models used to show how fractional parts are combined or separated?
How do we use models to construct algorithms for adding and subtracting fractions?</td></tr>
</table>

	for Fraction Game (Fraction Tracks) **The Role of Representations** (Cramer) **Bits and Pieces II** (Lappan)	
6	**FRACTION MULTIPLICATION** Modeling Multiplication of Fractions Multiplying Fractions with Manipulatives Moving from Models to Algorithms **Lamon** Chapter 11 **Minilesson DVD for Exploring Playgrounds** (Hersch)	How is multiplying or dividing whole numbers similar to multiplying or dividing fractions? How are multiplication, division, whole numbers, and fractions related? How can multiplication of a whole number by a fraction be modeled? How can multiplying or dividing fractions be modeled using area, length and set models? How is multiplication of fractions similar to repeated addition of fractions? What is the relationship between multiplication by a fraction and division? What is the relationship between the division of whole numbers and multiplication of fraction reciprocals? How do we use models to create algorithms for multiplying and dividing fractions? How can multiplication and division of fractions be used to represent and understand real world, and mathematical problems?
7	**FRACTION DIVISION** Modeling Division of Fractions Dividing Fractions with Manipulatives Moving from Models to Algorithms **Lamon** Chapter 11 **VDOE website**: 2014 SOL Institute Materials (see sample lesson for link) **Extending Children's Mathematics** (Empson)	
8	**"BASE TEN" FRACTIONS** Decimals and Percents % > 100 % < 100 The Case of Randy Harris **Using Cases** **Bits and Pieces III (**Lappan)	How are fractions and decimals alike and different? How can models help us understand the addition and subtraction of decimals? What type of physical models best represent decimals and percents? Why? Where do we see decimals and percents being used in our world? Why?
9	**RATIONAL NUMBERS AND THE REAL NUMBER SYSTEM** The Real Number System (Venn Diagram) The Case of Marie Hansen	When it is most appropriate to use a fraction, decimal or percent to solve a problem? Why? Why is it helpful to know how to convert between a decimal, a fraction, and/or a percent?

	Using Cases	Fractions, decimals, percents on a number line. How are proportions used to solve multistep ratio and percent problems?
10	**RATIO, RATES, UNIT RATES and PROPORTIONS** The Case of Janice Patterson **Using Cases**	What quantities should be compared? What type of comparison will give the most useful information? How can the comparison be expressed in different but useful ways?
11	**ADDITIVE vs. MULTIPLICATIVE THINKING (**RELATIVE vs. ABSOLUTE) Tree Problem Mr.Tall/Mr. Short Ratio Tables **Lamon** Chapters 1/ 3 **Minilesson DVD for Working with Ratio** **Tables** (Cameron) **NCTM YB** Activities	How can I determine if a relationship is additive or multiplicative? What are the characteristics of an additive relationship? A multiplicative relationship?
12	**DILATIONS / SCALE FACTOR** The Case of Marcia Green Monster Dog Activity or Growing Designs **Using Cases** **Flea-sized Surgeons**	What is a dilation and what effect does this transformation have on a figure? What is the significance of a scale factor of one? A scale factor greater than one? A scale factor less than one? How are coordinates and algebraic techniques used to represent, interpret, and verify geometric transformations?
13	**SIMILAR FIGURES** Rectangle Ratios	How can I tell if two figures are similar? What strategies can I use to determine missing side lengths and areas of similar figures? Under what conditions are similar figures

	NCTM YB Activities **AIMS-Proportional Reasoning** (Erikson)	congruent? In what ways can I represent the relationships that exist between similar figures using scale factors, length ratios, and area ratios? What strategies can I use to determine missing side lengths and areas of similar figures?
14	**PROBABILITY/DATA/STATISTICS** Baskets, Stats, Probabilities Catch and Release **AIMS - Statistics and Probability**	How do ratios help determine the likelihood of an event occurring? How do ratios help make decisions – is a game worth playing? What are my chances of winning? How can given comparison data be used to make predictions about unknown quantities? How do I use my knowledge of ratios to design a simulation that allows me to collect information to make predictions?
15	**FUNCTIONS** Ratio and foundation of slope Slope as Constant Rate of Change A Line on Pendulums **AIMS-Looking at Lines**	How are ratios represented in tables, graphs, equations, and diagrams? How are ratios used to represent and solve real-world problems? How do equations represent proportional relationships?

Sample Lesson Plan for Class 7:
FRACTION DIVISION

Textbook: Empson, S. & Levi, L. (2011). Extending children's mathematics: Fractions and decimals. Portsmouth, NJ: Heinemann

Materials: grid paper, tiles, fraction strips, cuisinaire rods, pattern blocks, materials to make number lines (sentence strips or adding machine tape), two color counters

Students will have read portions of YMW Chapter 7 (Using Efficient Computation with Minilessons) and Lamon Chapter 11 as part of the homework for previous class. For this class, they should review those and read VandeWalle Chapter 16 Section on Division (Conceptual Examples and Models, Answers That are not Whole Numbers, Developing the Algorithms and Addressing Misconceptions).

Essential Questions
How is multiplying or dividing whole numbers similar to multiplying or dividing fractions?
How are multiplication, division, whole numbers, and fractions related?
How can multiplying or dividing fractions be modeled using area, length and set models?
How is multiplication of fractions similar to repeated addition of fractions?
What is the relationship between multiplication by a fraction and division?
What is the relationship between the division of whole numbers and multiplication of fraction reciprocals?
How do we use models to create algorithms for multiplying and dividing fractions?
How can multiplication and division of fractions be used to represent and understand contextual, and mathematical problems?

Math Talk Warm-Up (20 minutes)

- Distribute the following question on a handout to students: Jay is making flowerpots. It takes ¾ of a package of clay to make 1 flowerpot. How many flowerpots can Jay make with 4 ½ packages of clay? Ask the participants to solve TWO ways. Encourage them to illustrate their thinking with words and pictures as they may be asked to share with the whole group (8 minutes).
- Walk around the room to see how students are approaching the problem.
- Sequence the sharing of materials in this order: solved using traditional division algorithm, explained the traditional algorithm in words, used repeated subtraction, used ratio tables, and used a drawing or model (examples of each type of model (area, length, set)) shown should be shared. There have been many models introduced in the classes prior to this, so hope to see a variety of model used.

Math Activity Fraction Division Equation Strings (1 hour) (materials adapted from Empson, S. & Levi, L. (2011). Extending children's mathematics: Fractions and decimals. Portsmouth, NJ: Heinemann and modifications available at http://www.pen.k12.va.us/instruction/mathematics/professional_development/index.shtml)

A great way to teach fraction division is to use equation strings involving progressively difficult problems (as we have seen in earlier mini lessons and in the YMW textbook). The problems listed below are the problems that will be used. They should be displayed on chart paper. Call attention to the fact that they are arranged in order of difficulty, so that as students move down the column, they can extend the reasoning used on one problem to solve the next, more challenging problem.

$$6 \div 2 = t$$

$$6 \div \frac{1}{2} = w$$

$$3\frac{1}{2} \div \frac{1}{2} = w$$

$$6\frac{3}{4} \div \frac{1}{2} = m$$

$$6\frac{3}{4} \div 1\frac{1}{2} = m$$

$$6\frac{3}{4} \div 1\frac{1}{5} = m$$

Explain to students that it is equally important for school students to move fluently among the different types of models that can be used and to choose the one best suited for the problem. Providing experiences for students to do these using story problems is an excellent way. So each of the problems above has been given a story, and a suggested model (or, in this case, a choice of models as the difficulty progresses).

** Sometimes when the answers do not provide a whole number, there will be a discussion of whether the answer is valid. For example in story problem four, it asks how many batches Francesca can make. The answer is 1 ½ batch. Some will argue that the answer is just 1 because you cannot make another whole batch. **

Story Problem One: It's Friday and Alex knows his mom has made brownies for him and his friend Brian. She cuts the pan of brownies into six equal sized portions. If Alex shares the brownies
with his friend Brian, how many can each have? The problem you are solving is: $6 \div 2 = t$.
Please use an area model to illustrate your thinking. You might want to try grid paper.

Story Problem Two: Carla has just returned home from the store with six dozen eggs. She must repackage them so that they are ready to make family-size omelets that require a ½ dozen eggs. How many omelets will she be able to make? The problem you are solving is: $6 \div \frac{1}{2} = w$. ***Please use a set model to illustrate your thinking. You might want to try two-color counters.***

Teaching Note: The idea that the answer to a division problem can be greater than the number being divided (or that multiplication can result in a smaller number) is counterintuitive. Students will only come to realize this understanding after many opportunities to visualize the impact of dividing (and multiplying) by a fraction less than 1. It is important to include this in part of your discussion of the problem as you are sharing solutions.

Story Problem Three: David wants to enter his team into a 3 ½ mile relay. Each runner must run a ½ mile leg. How many runners will David have to put on his team to complete the relay?

The problem you are solving is $3\frac{1}{2} \div \frac{1}{2} = w$. ***Please use a length model to illustrate your thinking. You might want to try a number line.***

Story Problem Four: Francesca has ¾ cups of flour left in her canister. She needs ½ cups to make a batch of cookies. How many batches will she be able to make with the flour on hand? The problem you are solving is: $\frac{3}{4} \div \frac{1}{2} = m$. ***Please use an area model or pattern blocks to illustrate your thinking.***

Story Problem Five: Greg is preparing medals for his Special Olympics team. Each medal requires half a yard of ribbon. He has 6 ¾ yards of ribbon. How many medals can he make? The problem you are solving is: $6\frac{3}{4} \div \frac{1}{2} = m$. ***Please use a length model to illustrate your thinking. You might want to try fraction strips or a number line.***

Story Problem Six: Hanna has 6 ¾ pounds of candy. She wants to separate it into bags of 1 ½ pounds each. How many bags will she have? The problem you are solving is: $6\frac{3}{4} \div 1\frac{1}{2} = m$. ***Use the model of your choice. Explain why you chose the model you did.***

Story Problem Seven: You have $6\frac{1}{5}$ gallons of gas left in your can. It takes 1 ½ gallons of gas to mow a lawn. How many lawns can you mow with the gas remaining? The problem you are solving is: $6\frac{3}{4} \div 1\frac{1}{5} = m$. ***Use the model of your choice. Explain why you chose the model you did.***

BREAK: (10 minutes)

Designing your Own Story (45 minutes total: 15 minutes for Group Work/5 minutes for each group to share out):

Divide the whole group into six small groups. Assign each of the group one of the equation from the string above. Ask each group to invent a story problem that asks the student to use a model other than the one used in the story problems we did together. They will be asked to provide a story for the problem. Solve it using a specific model with pictures and words. Explain why you think the model you used for the problem was the most appropriate? What model might not have worked as easily for your problem? Where do you see evidence of the algorithm is your model?

Mini Lessons for Operations with Fractions, Decimals, and Percents (40 minutes)
Participants will watch as students in a classroom participate in mini-lessons with fraction division. The facilitator's guide breaks each section of video clips into groups according to like

strategies, understandings, and/or misconceptions. The facilitator may choose to use some or all and choose applicable questions for the group to consider.

Exit Pass (5 minutes): Respond to the following question, paraphrased from the VandeWalle text, recorded on chart paper for the participants to read:

Fraction computation is too often taught as a series of rules or algorithms. We know that in order to develop effective instruction we must include meaningful contexts and examples, use appropriate models and manipulatives, value invented strategies, and explicitly address misconceptions.

Homework for next Class (5 minutes):
Complete the Opening Activity for "The Case of Randy Harris"
Read the Case of Randy Harris (be able to share the response to your question with the group when raised during whole group conversations)

Algebra and Functions for Mathematics Specialists

Functions and Algebra I is a 3-credit hour graduate mathematics course designed to prepare teachers with at least three years of classroom teaching experience to become school-based mathematics specialists. The course will develop a deep understanding of teaching algebra topics in the K-8 curriculum: variables, patterns and functions; modeling with and interpreting multiple representations; linear equations and equality, linear functions, slope as a rate of change and intercepts related to contextual situations. In addition, the course will introduce non-linear functions, including quadratics and exponentials. Through class discussions and class activities participants will connect these algebraic concepts to the related concepts that support primary and middle school arithmetic such as equality and basic computations. Attention will also be given to interpreting and assessing students' thinking.

Course Goals

Depth and sophistication of knowledge are demonstrated through the participants' ability to complete mathematical problems, discuss and explain the mathematical concepts through multiple representations, and connect the algebraic concepts to K-8 arithmetic topics. This course will focus on the following.

1. Develop skills and knowledge to use representations, algebraic reasoning, and generalization to support algebraic justification or proof.
   - Investigate and describe the real number system and its subsets. Understand and use the basic operations with integers. Understand and use the progression for showing how a truth relationship that holds for all numbers may be established, even though there are infinitely many numbers.
   - Use the components of valid mathematical arguments including the laws of arithmetic and justification to support generalizations.
   - Explain and use meanings of the interpretation and manipulation of variables, algebraic expressions, and algebraic equations.
   - Apply basic properties of arithmetic operations, e.g. the distributive property, to manipulate numerical and algebraic expressions and equations.
   - Represent and interpret algebraic functions: linear, quadratic, and exponential.
   - Solve linear equations and systems of linear equations in two unknowns.
   - Identify the solution set for a linear function.
   - Move fluidly and flexibly between discussing and making connections among contextual, verbal, graphical, tabular, and symbolic representations.

2. Develop and use an understanding of how children develop algebraic reasoning, mainly using case studies and analyzing student work.

Course Overview

In the elementary grades, mathematics has traditionally focused on developing arithmetic computations. This focus on computation continued when secondary school algebra was taught as a set of procedures. As a result, few connections were made between arithmetic and algebra and little attention is given to identifying patterns and using generalizations to develop mathematical arguments. The primary goal of this course is to prepare the mathematics specialist with the knowledge and skills to address the current disconnect between elementary school arithmetic and secondary school algebra. The course provides prospective mathematics specialists with the opportunity to study the conceptual connections between arithmetic and algebra and to develop a K-8 perspective on algebraic reasoning as well as the algebra tools needed for justification and proof. Students will gain an understanding of equivalence and relational thinking moving from an arithmetic context to an algebraic context, progressing from analyzing and describing patterns in a single variable to functional thinking that supports analyzing and representing change using two variables.

In the first half of the course, participants use, *Reasoning Algebraically About Operations* (RAO) supplemented with activities from *Thinking Mathematically: Integrating Arithmetic and Algebra In Elementary School* (TM). The mathematics activities and classroom cases in the texts enhance the study of arithmetic computations by articulating generalizations about the operations through expressing the generalizations using multiple representations words, symbols, drawing or diagrams, and physical manipulatives. Connections are made to the *Number and Operations course* and justifying the generalizations with representations and symbolically leading to the Laws of Arithmetic. This work begins with whole numbers, but the generalizations are extended to integers. A connection can be made to the course, *Rational Number and Proportional Reasoning for Mathematics Specialist*, by bringing fractions into these discussions. Instructor's supplement the Laws of Arithmetic by introducing the properties of equality. Instructors will need to pay attention to developing student's confidence to use variables as a tool for communicating algebraically.

In the second half of the course, participants use the book, *Patterns, Functions, and Change* (PFC) which focuses on developing the concept of functions. This study of functions begins with examining repeating patterns and growing patterns concrete and pictorially, and arithmetic sequences to see how they can be described symbolically as a relationship between the elements in the pattern or sequence and position of the element. Further development of the function concept investigates the mathematics connections among tables, graphs, symbolic notation, and contextual situations and how each of these representations reveals the notion of change. There is a heavy emphasis on understanding linear functions, but students leave knowing the difference between algebraic notation, table, and graph representations of linear, quadratic, and exponential functions, and the physical models or situations these functions describe.

Focus is placed on how to interpret a linear equation and how to solve a system of simple linear equations. While students in the cases in the *Numbers and Operations* course are seen considering negative numbers, the *Patterns, Functions, and Algebra* (PFC) text develop an understanding of the set of integers and operations with negative numbers. The characteristics of a linear function are well developed, but the instructor will need to supplement the PFC text activities to address negative and rational slope. .

Course Format and Key Activities

A variety of formats has been employed to teach The *Numbers and Operations* course. For example, it has been taught as two-week residential and commuter summer institutes with 54 hours of class time and significant in-class work and homework assignments and as a school-year course with 15 three-hour sessions in one semester or spread over two semesters. The timeframe in which the course is offered can impact participants experience and the instructors will need to plan accordingly. One benefit of the summer institutes is that students immerse themselves in the course and have the opportunity for additional collaboration with their peers and the instructors after class hours. This is particularly true for the residential institutes. However, in the summer, there are few if any relevant opportunities readily available to immediately do the mathematics with their students, to interview students about their understanding of the mathematics, or write cases based on their students. Instructors in the summer institutes use video of student interviews and bring samples of student work from other sources to provide opportunity for students to experience analyzing where students are in the developmental progression for number and make recommendations for next instructional steps

The instructional methodology includes small group and whole group discussions anchored in written and video cases of students mathematics thinking; cooperative group work around mathematics content and mathematics content pedagogy; and analyzing student interviews, student work, and cases from participants' practice. While developing the participants' mathematics content knowledge for teaching is the focus of the course it is just as important to include the case studies as a venue to deepen their understanding of how children make sense of the mathematics. The case studies also bring validity to teaching mathematics for conceptual understanding as well as for computational fluency. Class discussions about the mathematics as well as the cases become more robust as participants develop deeper understandings of the mathematics and the developmental progression of various mathematics topics. Course instructors bring explicit attention to the mathematics content, the developmental progression of the mathematics concepts, how children make sense of the mathematics, and which pedagogical moves afford students opportunities to become mathematical thinkers. Instructors intentionally model inquiry teaching throughout the course

Ongoing informal and formal formative assessment is an important component of the course. Instructors continually adapt the class activities and course projects to support participants to construct understanding and make connections to their classroom practice that deepen their understanding of how school students make sense of the mathematics. The course projects include maintaining a portfolio of the mathematics problem sets assigned for homework, completing two student interviews, analyzing two sets of student work, and maintaining a reflection journal throughout the course. Rubrics are provided for each project and writing is a component of each project. In addition, to the ongoing informal and formal formative assessments, there are three summative assessments, a midterm test, a cumulative final exam, and a final reflection synthesis paper.

Course Materials

Listed below are the primary student and instructor texts for the course. In addition, instructors will include supplementary readings such as NCTM journal articles.

Primary Student Texts

Schifter, D., Bastable, V., & Russell, S.J. (2008). *Developing mathematical ideas: Reasoning algebraically about operation's casebook.* Parsippany, NJ: Dale Seymour Publications: Pearson Learning Group.

Schifter, D., Bastable, V., & Russell, S.J. (2008). *Developing mathematical ideas: Patterns, functions, and change casebook*. Parsippany, NJ: Dale Seymour Publications: Pearson Learning Group.

Primary Instructor Resources

Carpenter, T.P., Franke, M.L., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary school.* Portsmouth, NH: Heinemann. (Includes DVD)

National Council of Teachers of Mathermatics. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM. (A student text in Leadership I.)

Schifter, D., Bastable, V., & Russell, S.J. (2008). *Developing mathematical ideas: Reasoning algebraically about operation's facilitator's guide.* Parsippany, NJ: Dale Seymour Publications: Pearson Learning Group. (Includes DVD)

Schifter, D., Bastable, V., & Russell, S.J. (2008). *Developing mathematical ideas: Patterns, functions, and change facilitators guid*e. Parsippany, NJ: Dale Seymour Publications: Pearson Learning Group. (Includes DVD)

Van de Walle, J., Karp, K., and Bay-Williams, J.M. (2010). (7th edition). *Elementary and middle school mathematics: Teaching developmentally*. New York, NY: Pearson Education (Allyn & Bacon). (A student text in Leadership I.)

Instructors can find additional information from the developers and various implementers of the DMI materials at http://www2.edc.org/cdt/dmi/dmiless.html.

Supplementary Readings

Cleaves, W.P. (2008). Promoting mathematics accessibility through representations: Jigsaws. *Mathematics Teaching in the Middle School*, *13*(8), 446-452.

Knuth, E.J., Choppin, J.M., & Bieda, K.N. (2009). Examples and beyond. *Mathematics Teaching in the Middle School, 15(*4), 206-211.

Milina, M. & Ambrose, R. (2006). Negotiating the meaning of the equal sign. *Mathematics Teaching Children Mathematics, 13*(3), 111-117.

Mueller, M., & Maher, C. (2009). Convincing and justifying through reasoning. *Mathematics Teaching in the Middle School, 16(*2), 108–16.

Suh, J. (2007). Tying it all together: Classroom practices that promote mathematical proficiency for all students. *Mathematics Teaching Children Mathematics, 14*(3), 163-169.

Van Dyke, R. & Tomback, R. (2005/2006). Collaborating to introduce algebra. *Mathematics Teaching in the Middle School, 10(*5), 236-242.

Course Outline: Topics and Essential Questions

The overview displayed in Figure 5 presents the scope and sequence of topics in the Functions and Algebra I course taught in 15 weekly 3-hour class sessions. The overview identifies the topics and essential questions for each class and resources used to support each class. The course textbooks are noted in the outline as follows.

PFC: *Developing Mathematical Ideas: Patterns, Functions, and Change*
RAO: *Developing Mathematical Ideas: Reasoning Algebraically About Operation*
TM: *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School*

Figure 5. Algebra and Functions Course: Overview of Topics

Class	Topics/Resources	Essential Questions
1	Generalizations Developing mathematical arguments Developing mathematical definitions Communicating a generalization Using variables to write expressions RAO Chp 1 TM Chp 7	What does it mean to generalize? What is a mathematical conjecture? What comprises a mathematical argument that is always convincing when every number or case cannot be checked? What is the role of representation in developing mathematical arguments? What are the levels of justification? How do models such as visual images, story contexts, number lines, concrete materials, and variables help to express and justify generalizations?
2	Set of Real Numbers The progression from arithmetic reasoning to algebraic reasoning Generalizing arithmetic relationships Inverse property for addition	What subsets make up the set of Real Numbers and what is the relationship between the subsets? How does a number line model the set of Real Numbers?

	Introduce the transitive property Introduce the substitution property of equality RAO Chp 2 TM Chp 4	What constitutes proof at various levels of a developmental progression? What role does mathematical structure and patterns play into proof? What is the role of the properties of arithmetic in various ways of performing calculations with real numbers? What generalizations underlie different computational strategies for addition and subtraction?
3	Operating with integers Equality and the equal sign Introduce inequality Relational thinking Student interview Commutative property Associative property RAO Chp 3 TM Chp 2 and Chp 3	What arguments justify that switching the order of addends results in the same sum and that switching factors results in the same product? What constitutes a helpful concrete or pictorial model? What pattern or regularity occurs when switching the order of numbers in a subtraction or division problem and how can the patterns be explained? What understanding is necessary to make sense of the equal signs, what are some common misconceptions that surface in students' thinking? In what situations are negative numbers useful? How does student thinking about making arguments develop across the grade levels?
4 Sample lesson plan following the overview of classes	Adding and subtracting with integers Algebraic thinking Introduce the concept of absolute value Solving linear equations Tools: number line, charge model, algebra tiles RAO Chp 4	What are the strengths and limitations of two physical models, number line, and charge model, for operating with negative numbers? What ideas about what it means to order and compute with integers need to be modified when the number system is extended to include 0 and negative numbers? How can the relationship between addition and subtraction be used to make sense of subtraction with negative numbers?
5	Inverse property of addition and multiplication Identity elements for multiplication and addition	What is the role of the inverse relationship, if a+b=c then c-a=b and c-b=a, in solving equations? What properties support symbol

	Multiplication and division with integers RAO Chp 5 TM Chp 6	manipulation when solving equations? How are the roles of 0 in addition and 1 in multiplication the same? How are they different? How do the fundamental mathematical ideas that emerge from relationships in number sentences support students in developing more flexibility in arithmetic and in developing algebraic thinking?
6	Greatest common factor and least common multiple Distributive property Polynomials: adding, subtracting, and multiplying Exponents: multiplying monomials and binomials RAO Chp 6 and Chp 7 RAO Math Activity pgs. 270-273 TM Chp 8	What does it mean to understand the Distributive Property? How is multiplication with whole numbers related to multiplication with polynomials? What are the ways to explain what happens when multiplying or dividing two negative numbers? What is the relationship between a number and its factors? What is the role of exponents? How are the Laws of Arithmetic related to the Field Properties of Mathematics?
7	Arithmetic sequences Analyzing and describing concrete and pictorial growing patterns (figural sequences) and arithmetic sequences Verbal and algebraic expressions for the nth term Introduce terms relationship and function PFC Chp 1 TM Chp 5	How can concrete and pictorial growing patterns (figural sequences) and arithmetic sequences be represented and analyzed in tables? What number sequences or figural sequences can be described as a functional relationship? How can verbal descriptions, tables, pictorial/concrete representations, and symbolic representations be used to express a generalization for a relationship between two quantities?
8	Coordinate plane: axes, intercepts, origin, ordered pair (x,y), quadrants, scale, plotting points Input/ output Independent/dependent variables Domain/range Features of a graph: intercepts, continuous vs. discontinuous, linear vs. non-linear, increasing vs. decreasing	Do graphs from a particular situation have identifying features? Will analyzing different representations, that is context, concrete/pictorial, tables, and graphs, for the same situation reveal differing perspectives about the relationship between two quantities? What kind of relationships can be expressed as a linear function? What are the generalizations and arithmetic

	PFC Chp 2	expressions that describe linear functions?
9	Linear functions Co-variation Recursive reasoning versus functional reasoning Rate of change Slope as rate of change (positive slopes) x- and y- intercepts Writing algebraic equations for the n^{th} case $y=mx+b$ $f(x)$ notation PFC Chp 3	How do different representations, that is pictorial/concrete, tables, and graphs, lead to algebraic representations for the nth case? What are the identifying features of a linear function and how are those characteristics displayed in multiple representations: pictorial/concrete, table, graph, and algebraic equation. What is the rate of change in a linear relationship and how is it related to proportional reasoning? What contextual or real-world situations can be modeled with a linear function? What does it mean to engage in functional thinking?
10	Slope as rate of change: negative and fraction slopes, 0 and undefined slopes Parallel lines Systems of linear equations PFC Chp 4	How is a constant rate of change displayed in various representations of a situation: graphs, algebraic equations, tables, and unconventional representations created by students? What can be learned by examining the rate of change or slope for linear functions and from pairs of linear functions? When two lines intersect what does it mean in terms of the contextual situation, the corresponding table and graph, and algebraic equations?
11	Direct variation Proportional Reasoning and linear functions Constant of variation $y = kx$ PFC Chp 5	What determines whether the change is proportional or non-proportional growth? When a function represents direct variation how can it be interpreted in terms of the contextual situation, the corresponding table and graph, and an algebraic equation? What is the relationship between the non-constant rate of change and non-linear functions?
12	Quadratic functions PFC Chp 6 Fixed perimeter and changing area fencing problem	How do the graphs of linear functions compare to nonlinear functions? How does the rate of change impact the shape of the graph? What does a quadratic relationship look like in a context, table, and graph? What are the characteristics of the graph of a

	Handshake problem	quadratic function? How are quadratic relations represented with algebraic notation?
13	Exponential function PFC Chp 6 Towers of Hanoi problem Koch Snowflake fractal problem	What are the characteristics of an exponential function? How can a function be written for an exponential relationship? How can patterns of change be used to determine functions? How is the class of Polynomial Functions used to classify particular functions including linear, quadratic, and cubic?
14	Numberless graphs PFC Chp 7 and Chp 8 Van Dyke, R. & Tomback, R. (2005/2006). Collaborating to introduce algebra. *Mathematics Teaching in the Middle School, 10(*5), 236-242. (Numberless graphs)	How can a graph represent both quantity and change in quantity? What are the relationships between the shape of a graph and the context or phenomenon?
15	Painted Cube Problem (RAO Pgs. 309-310) Final Exam (1.5 hours)	How is the class of Polynomial Functions used to classify particular contexts including linear, quadratic, and cubic?

Sample Lesson Plan for Class 4: Functions Algebra I
Introducing the set of real numbers and addition and subtraction with integers.

Textbook: RAO Chapter 4 Expanding the Number System

Materials: cubes, two-color counters, chart paper, and markers
Students read RAO Chapter 4 Cases 15-19 for homework prior to class.

Essential Questions

- When the number system is extended to include 0 and negative numbers what needs to change in the way numbers are considered?
- What ideas about what it means to order and compute with integers?
- What are the strengths and limitations of two physical models, number line and charge model, for operating with negative numbers?
- How can the relationship between addition and subtraction be used to make sense of subtraction with negative numbers?

Math Talk Warm-Up (15 minutes)

- Write on chart paper: a + b = (a-c) + (b-c)
- Pose a question to the class: Is this generalization true or not. Justify your thinking in two different ways; you can use words and pictures. The students are not expected just to know the distributive property but rather the expectation is to reason about the situation. (5 minutes)
- Walk around the room to see how students are attacking the problem. When sharing out justifications in this order: used numbers to test the equation, reasoning, pictorial or concrete, symbolic.
- During the discussion refer to the levels of justification, look for the opportunity to discuss any of the Laws of Arithmetic but do not force this discussion, and evaluate students' confidence and comfort level with symbolic notation.

Debrief Student Interview about Equality Project Assigned in Class 2 (30 minutes)

Work in the same group of 3 that have been collaborating on developing their student interviews; each person has 8 minutes to share and receive feedback from the group

- 5 minutes each to share what you learned about your student and what you recommend as next instructional steps.
- Group members can pose questions for clarification or to help the presenter consider a different perspective.

Math Activity Introduction to Negative Numbers: (15 minutes)

- Generate a class list of situations in which negative numbers are used. The list does not need to be an exhaustive list but collect situations until the variety of different situations listed that will be useful in modeling the operations with negative numbers.
- Collect the ideas students share on chart paper.
- Ask the table groups to consider the class list to see how they might sort and classify the things that have been listed.

Math Activity Introduction to The Set of Real Numbers (40 minutes)

- Start with a number line, -10 to 10 and scaled by units on chart paper. Ask students to write down everything they can remember about the set of real numbers and also, what they notice about the number line as they think about the set of real numbers.
- Have table groups ~~to~~ take turns sharing one thing and go around the room until all ideas are collected. Record the responses on chart paper. Use the students' responses to assess their prior knowledge. Use the class responses to bring out terms and ideas such as integers, rational numbers, whole numbers, natural numbers, opposites, distance from 0; a number represents a point on the number line, and there is an infinite sent of points on the number line.
- Watch RAO Chp 4 DVD (8 minutes), ask students to make notes on how these young children are making sense of the number line and how they are making sense of negative numbers. After the video ask table groups to discuss their notes from the video and to consider the pre-video discussions about the number line made by the class.

- Extending Case 18. Ask the class to discuss the meaning of (–a). Then pose the question, "Which is greater *a* or the –*a*? for small groups to consider. Walk around and notice how groups are thinking about the question. Select several ideas to share with the class such as reasoning, number line, tables, etc. Then use the responses to discuss how to write the solution using algebraic notation: if a>0 then a>-a and if a<0 then a<-a but if a =0 the a= -a.
- Bring out that the negative numbers are a set within a well-organized Real Number system. Take some time to develop a graphic organizer that shows the hierarchal inclusion of the subsets of the real number system. Irrational numbers are addressed in the second algebra course and should be given limited attention at this point

Models for Working with Integers (20 minutes)
Prepare students to use the charge model and the number line model to represent adding with integers.

- Introduce and demonstrate the charge model for modeling addition and subtraction with integers. Review the Additive Inverse Property a+-a=0.
- Introduce the number line as a tool for modeling addition and subtraction with integers

Math Activity RAO Chp 4 Addition and Subtraction with Integers (25 minutes)
Use RAO-FG HO Math Activity page 145 to have students work on questions 1. It may be helpful to suggest that students use the numbers 3, -3, 7, and -7 and the models for addition and subtraction.

- Groups work for 15 minutes and then have some students share. This activity explores the commutative property while giving students a chance to use a number line and charge model to add and subtract integers.
- Instructors use this time to work with individuals or small groups who are still making sense of adding and subtracting integers.

Math Activity RAO Chp 4 Comparing and Ordering Integers (20 minutes)

- Ask the class to share how they know that 9 is greater than 7. Collect the responses on chart paper. Continue collecting until no new ideas emerge.
- Small groups work on question 3, compare integers using a number line and charge model and then use the class list to which models hold up, can be generalized for comparing any integer to another integer.
- Whole group discussion: Which ideas support 7>-9; -9>7; or does not apply to negative numbers
- Use the class responses to develop the generalization a< b if there is a c such that a+c=b.
- Introduce the concept of absolute value

Homework for Class 5
1. Read RAO Chapter 5 Doing and Undoing, Staying the Same.
    - Focus Questions, page 181, consider questions 1-3 and develop a written response.

Algebra for Middle School

Algebra for Middle School is a 3-credit hour graduate level mathematics course designed to contribute to the preparation of teachers with at least three years of classroom teaching experience to become school-based mathematics specialists. This course is intended to follow *Algebra and Functions* and to be a second algebra course to help deepen the understanding of teachers specifically preparing to serve in the middle grades.

The course content will help participants develop a deeper understanding of algebra topics in the grades 6-12 curriculum: variables; patterns and functions; modeling with and interpreting multiple representations; linear and non-linear functions (including but not limited to polynomial, exponential and rational functions); systems of linear equations; and number theory related to the real and complex number systems. The emphasis will be placed on activities that require students to demonstrate an understanding of the concepts and that focus on pedagogical techniques appropriate for middle school students.

Course Goals

Understanding will be demonstrated through each participant's ability to complete mathematical problems; discuss and explain the mathematical concepts; model problems with graphing calculators and other technologies or manipulatives; and to connect the mathematical concepts within mathematics and across the curriculum. The course will focus on the following:

- Develop skills and knowledge to use representation, algebraic reasoning, and generalization to support an algebraic proof.
- Use valid mathematical reasoning and the properties of the real number system to support generalization.
- Manipulate algebraic expressions and equations (including numerical and symbolic methods, algebra tiles or other manipulatives, and the graphing calculator).
- Model and interpret algebraic functions, including but not limited to linear, quadratic and other polynomial functions, exponential, and rational.
- Solve linear and non-linear equations and systems of equations.
- Use linear combinations to interpret and solve problems.
- Use finite differences to determine if a relationship is linear, quadratic, or cubic (or none of these) and to develop the appropriate equation that models the data.
- Solve problems with non-real solutions, either imaginary or complex.
- Solve quadratic equations with a variety of methods including graphing, factorization, and the quadratic formula.
- Apply the properties of arithmetic to a different number system (to better understand these laws).
- Use multiple representations to solve and present solutions to real problems and rich tasks; these should include physical or pictorial, verbal, tables or charts, graphical, and symbolic representations.

- Develop an understanding of how students learn mathematical concepts, using case studies and discussions.

Course Overview

The mathematics curriculum in the middle grades (6-8 or even 5-9) poses unique challenges for the instructor, and especially the mathematics specialist. Children enter middle school with diverse backgrounds and thought; some are at a concrete level of arithmetic understanding while others are already abstract thinkers prepared to generalize results. To be able to assist teachers in dealing with this range of student performance, the effective specialist at this level needs a deep and full understanding of the number system and properties that lead to algebra, as well as knowledge of more advanced algebraic topics and higher order functions that will be covered in high school mathematics. Of most importance is the ability to connect topics across the mathematics curriculum and incorporate strategies to reach each student. Rote learning of algorithms or memorized rules will not suffice.

The first algebra course in the specialist program (that precedes this one) addresses algebraic thinking and algebraic content of elementary school mathematics, how it connects to algebra, and also addresses of the key ideas covered in a first-year algebra course (e.g. linear functions). This course is intended to extend and deepen the understanding of these topics and develop mathematics concepts typically covered in a first- or second-year algebra course.

Each class session should include two distinct parts. Mathematics activities and problems will be used to introduce and/or revisit algebraic topics, with a focus on clarifying and strengthening student understanding. In addition, case studies and pedagogical discussions will emphasize how students make sense of algebraic concepts. Throughout the class activities, participants should be required to reflect on the value of the activities and if or how they might be best utilized in their home schools.

This course does not rely on one main textbook as some of the other courses do, but rather uses materials gathered from several resources listed in the resource section.

Course Format and Key Activities

This course can be taught in a variety of formats, including a two-week summer institute with 54 hours of class time and significant daily homework assignments and readings, or a regular semester course with 15 three-hour sessions. Varying formats provide different advantages for the participants. In a summer institute, especially if it is residential, students are immersed in the mathematics experience and have fairly unlimited opportunities for group work, in-depth discussions, and shared homework. In the typical semester course, there are opportunities to practice what they learn in their home schools and bring student work samples to class to compare and analyze.

All of the specialist mathematics courses have been developed with a student-centered approach. Students work in groups to do mathematics, discuss their understandings, and frequently reflect on what they have learned. This model makes use of rich tasks and problems, group activities, case studies, and group presentations of work. Course instructors facilitate the discussions and bring attention to important mathematical concepts, and provide brief tutorials and background information as needed. Students, with the assistance of instructors, tackle meaningful problems and engage in rich discussions about the mathematical content. This should result in a deeper understanding of the concepts and how they connect to other mathematical topics. Students are led to relate these understandings back to school mathematics and to their own students.

In the course overview, the topics and activities are listed according to general categories. The instructors may choose to mix these up somewhat and return to topics to expand them throughout the course. Several main threads which are intended to be carried out throughout the course are as follows:

- The use of rich tasks and problems as a method of introducing or clarifying key mathematical concepts, and to promote mathematical thinking as opposed to rote learning (at least one rich task in every one or two class sessions).
- The use of multiple representations to solve and present problems – including verbal, pictorial, physical (models), graphical, and symbolic representations (and in some cases trial-and-error or collecting data).
- The increasing use of algebraic representations to model real problems.
- Emphasis on group solutions and presentations rather than just isolated individual work.

Course Materials

Listed below are the primary student and instructor texts for this course. (Note: These are the books which were purchased and used in our classes. Most of the problems and activities are widely available, and different texts could be selected to offer similar material).

Primary Student Resources

Cooney, T., Beckmann, S., & Lloyd, G. (2013). *Developing understanding of functions, Grades 9-12.* Reston, VA: NCTM.

Driscoll, M. (1999). *Fostering algebraic thinking: A guide for teachers grades 6-10.* Newton, MA: Educational Development Center, Inc.

Hyde, A. (2009). *Understanding middle school math.* Portsmouth, NH: Heinemann.

Smith, M., Silver, E., & Stein, M. (2005). *Improving Instruction in algebra: Using cases to transform mathematics teaching and learning.* New York, NY: Teachers College Press.

Additional Instructor Resources

Lund, C. & Andersen, E. (1992). *Graphing calculator activities; Exploring topics in algebra 1 and 2.* Boston, MA: Addison-Wesley Publishing Company, Inc.

Mason, J. (2005). *Developing thinking in algebra.* United Kingdom*:* The Open University.

Murdoch, J., Kamischke, E., & and Kamischke, E. (2010). *Discovering advanced algebra: An investigative approach, 2nd Edition*. Oakland, CA Key Curriculum Press.

Supplemented with readings and activities from NCTM publications, including *The Mathematics Teacher* and *Mathematics Teaching in the Middle School.*

Course Outline: Topics and Essential Questions

The overview displayed in Figure 6 presents the scope and sequence of topics in the *Algebra for Middle School Specialists* course taught in 15 weekly 3-hour class sessions. The overview identifies the topics and essential questions for each class and resources used to support each class.

The course textbooks are noted in the outline as follows.

DEF: *Developing Understanding of Functions.*
FAT: *Fostering Algebraic Thinking.*
UMSM: *Understanding Middle School Math.*
IIA: *Improving Instruction in Algebra: Using Cases to Transform Mathematics Teaching and Learning.*
VAF: *A Visual Approach to Functions.*
GCA: *Graphing Calculator Activities.*
DAA: *Discovering Advanced Algebra.*

The pedagogical class discussions should be partly based on assigned readings, which can be chosen by the course instructors. Some suggested readings include the following:

DEF – Any or all chapters
FAT – Any or all chapters
UMSM – Introduction, Chapters 1, 2, 3, 5.

Each class is designed for three hours. Assigned readings and discussions, as well as midterm and final exams, are not listed and should be fit into the curriculum accordingly.

Figure 6. Algebra for Middle School Course: Overview of Topics.

Class	Topics/ Resources	Essential Questions	Some Suggested Mathematical Activities
1	Using rich tasks	What does it mean to	• Assign a rich task to

	Multiple representations Developing mathematical arguments Communicating a generalization Introduction to graphing calculators Review and extension of graphing linear functions	generalize? What is the role of representation in developing mathematical arguments? How do models such as pictures, concrete models, number lines, graphs and symbolic expressions help to justify generalizations? What are the attributes of a linear function? What can be learned by examining the slope or rate of change of a linear function? Where are the intercepts found in a graph of a linear function, and what do they mean?	groups (e.g. The Checkerboard Problem) to brainstorm, solve, and present to the class. Groups should use multiple representations. • Brainstorm in small and whole group – the value of rich tasks in middle school instruction. • Mini-lesson for graphing calculators – introduce basic keystrokes as some will be unfamiliar, and proceed to graphing function keys. • Practice lab for graphing calculators – assign a set of questions for basic keystroke practice as well as an activity involving the graphing of linear functions (e.g. *Using Graphs to Introduce Functions*, NCTM, *The Mathematics Teacher*, 2003.
2	The real number system Number theory, including factors, primes, multiples, and the part they play in algebraic thinking Examining infinite sets Numberless graphs Algebraic habits of mind – doing and undoing	What is a mathematical conjecture? How can a mathematical argument be convincing when every possible case cannot be checked? What is an infinite set? What are some infinite sets included in the real number system? What are factors and multiples and how are they connected?	• Activity related to factors and factorization, including prime factors (suggested, *The Nu Function*, FAT, p. 55). • Activity to familiarize participants with infinite sets such as evens, odds, multiples of 5, perfect squares, etc. (suggested, *Number Fun*, FAT, p. 61). You can use primary cardstock strips to list the infinite sets for groups. • Solving problems using the first algebraic habit of mind “doing and undoing”

		What is a prime number?	(e.g. *Sneaking up the Line*, FAT, p. 35). • Assign a set of number theory, thought-provoking questions. • *The Stretching Problem* (rich task), FAT, pp. 54-55. • *Sums of Consecutive Integers*, FAT, p. 75)
3	Continue graphing calculator instruction Comparison of linear and non-linear functions Examine a variety of functions Absolute value functions	What are domain and range? Why do many of our rich tasks and real problems have limited domain or range? What is the difference between the graph of a particular situation and the entire graph of the associated function? What kinds of situations can be expressed as a linear function? What is different about a non-linear function? What are the characteristics of an absolute value function? What does it have in common with linear or non-linear functions?	• Assign a set of situations. Groups determine if each relationship described is linear or non-linear, or if no relationship exists. Groups present results, class discussion. • More graphing calculator instruction and practice (suggested, *Using Graphing Calculators to Model Real World Data*, NCTM, *The Mathematics Teacher*, 2004). • *A Continuum of Rich Tasks.* This video is a great resource from former NCTM president Glenda Lappan. It shows a related rich task for each grade level 3 – 8. Suggested that class watch each grade level, and stop the film for them to try each problem. At the upper-grade levels, assign the task and allow groups time to do the problem and present their results with multiple representations. Then compare their findings with the video clip. • A brief review of the concept of absolute value.

			Have groups generate tables of values for absolute value equations and graph. Compare to results on a graphing calculator. Discuss the V-shape and what it means.
4	Fundamentals of functions, including graphs, tables, and verbal descriptions Scatterplots Best fit lines The KWC approach to problem solving (should follow an assigned reading from UMSM Intro) Algebraic habits of mind – Building rules to represent functions (FAT)	What are the big ideas related to functions? What is covariation or rate of change and how does it relate to functions? What is the input-output relationship of a function? How is the domain related to the range? How can we represent a function in multiple ways? What is a scatterplot? Bivariate data? How can we express a relationship with a line of best fit?	• A lab activity that requires collection and interpretation of data (suggested, *Functions and the Volume of Vases,* NCTM, *Mathematics Teaching in the Middle School,* May, 2012). Unusual vases or containers can be found in thrift shops or dollar stores. • Discuss the KWC strategy from readings ("What do you KNOW for sure? What do you WANT to know? Are there any special CONDITIONS?) • Apply the KWC approach to a selected group task or problem (suggested, *The Tall Tale of the Two Texas Ranches,* UMSM, p. 8). Solve and present findings. • Set up various data collection stations around the classroom – this could be done in previous class session or during breaks). At each station, participants record or measure personal data. Some possible examples include height, shoe size, years of teaching experience, distance traveled to class, arm span, circumference of wrist or

			head, etc. Each set of data will represent one possible variable. Make sure all students are recorded at all stations. • Scatterplot Activity – Allow each group to choose 2 variables (data sets) to compare. Data sets can be copied and used more than once, but no group should compare the same two variables. Groups should examine the data, make decisions about best presentation, graph the bivariate data on a coordinate plane, analyze the results, and determine if there is a relationship. If so, estimate a line of best fit for the graph. Present results. • Informally discuss positive and negative relationships, correlation, and so on. More work on formal linear regression will be included in the probability and statistics class.
5	Modeling mathematical situations with algebra tiles or other manipulatives Linear combinations Modeling linear problems Systems of linear equations Algebraic habit of mind – abstracting from computation (FAT)	What are the strengths and weaknesses of modeling equations with algebra tiles? What is the relationship between a set of linear combinations and a system of linear equations? How can linear combinations and the associated computational	• Introduce algebra tiles or other manipulatives. Assign problems to be modeled and presented by groups. • Rich tasks involving linear situation (e.g. *Chocolate Math*, UMSM or *Tiling Garden Beds*, FAT, p. 135). • Activity in which participants are required to calculate various linear combinations related to

		shortcuts lead to generalization and abstraction of a mathematical situation? How can modeling linear problems lead to generalization and abstraction of the situation posed? When two graphed lines intersect, coincide, or are parallel, what does it say about the contextual situation/s represented by the lines?	specific problem situations (e.g. FAT, pp. 66-67). • Mini-lesson on solving systems graphically and algebraically. • Case Study – *Examining Linear Growth Patterns*, from IIA, pp. 8-31.
6	Examine characteristics of various polynomial functions including linear, quadratic, and cubic. Model polynomial equations with algebra tiles and manipulatives. Make connections from geometric models to linear, quadratic, and cubic equations.	What is the difference in the general shape of the graphs of various polynomial functions? What end behavior is characteristic of the graphs of various polynomial functions? What do the graphs of various polynomial functions indicate about the related problem situations? What is alike/different about various polynomial functions?	• Follow up discussion after rich tasks solved previously (can also refer to the Glenda Lappan slides); make sure participants see the connection of 1-dimensional models to linear functions, 2-dimensional models to quadratics, and 3-dimensional models to cubic equations. Emphasize the physical connection of a geometric figure to the resulting equation (rather than algebraic or numerical methods of writing the equation). • Rich task – Brainstorm first. "What kind of function do you think will result? Then assign

			problem or task for groups to solve and present with multiple representations (e.g. *The Skeleton Problem,* adapted from Mathematics Assessment, p. 117). • Case Study – *Examining Nonlinear Growth Patterns,* IIA, pp. 32-49.
7	Continued from previous class- Examine characteristics of various polynomial functions including linear, quadratic, and cubic. Model polynomial equations with algebra tiles and manipulatives. Make connections from geometric models to linear, quadratic, and cubic equations. Also – Use graphing calculators to graph and compare polynomial functions. Examine zeros, intercepts, end behavior, etc.	What connections can be made between a given problem situation and components of the associated polynomial graph (e.g. intercepts, zeros, etc.)? What are the general characteristics of a linear, quadratic and cubic equation? How are they alike? Different?	• Rich task – suggest *Painted Cube Problem* or another task which yields different categories of functions (this one can be found in many resources, including FAT and DTA). Have groups build cubes with dimensions 3x3x3, 4x4x4, 5x5x5, and then extrapolate the data for cubes of side length 5, 7, and n. They should build, record data, draw, and graph results, and, if possible, represent the results symbolically. • Use the graphing calculators to graph equations of linear, quadratic and cubic equations. Compare the graphs. Discuss. Give mini-lecture as needed to fill in missing points. • Assign problem sets to graph. Have students identify the zeros and intercepts. Discuss their connection and the solutions.
8	More formal	How does the rate of	• Graphing calculator lab

	examination of quadratic equations: Solve algebraically Solve by transformations Solve by graphing Solve real-world problems requiring quadratic models Use pertinent vocabulary such as descending order, coefficient, horizontal and vertical shifts, domain, range, maximum, minimum, etc. Introduce the complex number system and imaginary numbers.	change affect the graph of a quadratic equation? If we shift a graph horizontally or vertically, what can we say about intercepts, the rate of change? If we stretch or shift a graph, what can we say about the intercepts, rate of change? What are the ways to represent a quadratic equation symbolically? What are the different ways to solve a quadratic equation? How does the quadratic formula represent a generalized solution for any quadratic equation?	activity – Many resources are available with standard textbook materials and graphing calculator resources (e.g. GCA and VAF). Make sure to include examples with leading coefficients of 1, less than 2, and greater than 1. • Factoring with Algebra Tiles – Model, draw and represent answers symbolically. • Activity – *Find the Number of Real Roots by Graphing* (GCA, Activity 20). Use graphing calculators to answer *questions*. • Mini-tutorial on quadratic equations. Walk participants through a few sample problems, solving some by factoring, some with quadratic formula, and compare the results to graphs. • Mini-lecture on imaginary number unit *I*, also powers of *I*, and complex numbers. • Assign practice problems.
9 Sample Lesson Plan Attached	Examine exponents and their general properties Explore exponential functions Compare exponential graphs to graphs of polynomial functions Use graphing calculators to examine exponential functions	How do we “undo” squaring or cubing a number? What does an exponent represent? What are the characteristics of an exponential function? What is the nature of the growth or decay of an exponential function?	• Rich task – *Weighing Meat* (FAT, p. 75) – gives a different look at exponents, uses powers of3; an alternative activity might involve binary numeration. • Experimental activity which can be modeled by an exponential equation (suggest *Paper Folding).* • Problem set on exponential growth and decay (VAF,

		Why does the graph of an exponential function have a restricted range?	can select pages from 68-87). • Rich task *Modeling Half-Life with Skittles or M&M's* (found in many resources). Participants shake candies from small paper cups and record and throw away number with letter facing up. Continue till all are gone. Graph and analyze results. • Extend the concept of a transformational approach to graphing to exponential functions (shifts, shrinking, stretching).
10	Continue exponential functions from Class 9 Also- Introduce concept of logarithm Explore exponential growth of investments		• Mini-tutorial on logarithms (basic concept).Increase logarithmic expression as an inverse operation to an exponential expression, so if $3^4 = 81$, then $\log_3 81=4$. Include other examples of inverse operations that could apply to school math. • Rich task – suggest *Tower of Hanoi* (found in many resources). • Real life example of a logarithmic situation (e.g. the Richter Scale for earthquakes is a logarithmic scale, base 10) • Sample problems and/or activity sheets (can select from VAF, pp. 90-112) • Use graphing calculators to explore exponential equations (suggest GCA, pp. 27-30).

11	Direct and inverse variation Comparing exponential and logarithmic equations	What characterizes proportional growth? When a function represents a direct variation, how is that apparent from the context, the chart of values and the graph? When a function represents an inverse variation, how is that apparent from the context, the chart of values and the graph? How do the graphs of exponential functions compare to those of logarithmic functions?	• Assign a set of simple logarithmic equations to be completed on the graphing calculator (e.g. GCA, pp. 31-31) • Compare results to exponential examples from the previous day. As participants present their findings, make sure that connections are made, and appropriate vocabulary introduced as needed. • A brief introduction to direct and inverse variation, including k, the constant of proportionality. • Assign a set of problem situations and ask groups to determine if each represents a direct proportion, an inverse proportion, or neither. Groups present multiple representations to support their claims.
12	Examine assortment of functions. Characterize similarities and differences. Examine effective ways to help students understand functions. Solve problems that deal with time, distance, speed, and velocity.	What are the distinct characteristics of each type of function we have examined? For each type what can we say about the domain, range, intercepts, end behavior? What relationships exist between time and distance? How is velocity related to time and distance?	• Case Study – *Interpreting Graphs of Time versus Speed*, IIA, pp. 64-81. • Assign projectile problems (e.g. VAF, can select from pages 113-141) • Laboratory – Use CBL's or CBR's to gather data on distance, speed, time, and to evaluate the related equations (e.g. *Walk the Line*, *Match the Graph*, *Ball Drop*, etc.). Many resources available on Texas Instruments site, also NCTM. • Assign mixture of

			problems to be graphed using transformations, following patterns learned in earlier class sessions. Groups present results.
13	Inverse variation and the hyperbola Rational functions Asymptotes End Behavior of functions	How is the graph of a hyperbola related to inverse proportion? What are the characteristics of a hyperbola? What is the effect of an asymptote? How can an asymptote be determined for a specific function? What is different about a rational function and all the other functions studied previously?	• Use graphing calculator activity to help participants see that the graph of an inverse variation will be a hyperbola. Introduce appropriate vocabulary as needed. (suggest GCA, pp. 19-20 and 21-22). • Rich Task – *The Breaking Point* (available in a number of resources including DAA). Groups gather data using small paper cups of beans supported by varying lengths of dry spaghetti. Results approximate a rational function and should be presented with multiple representations.
14	Generalizations Finite Differences Review of real number properties Modular arithmetic	Do graphs of particular situations have identifying characteristics? How can finite differences be used to identify whether a function is linear, quadratic, or cubic? How can finite differences be used to write an appropriate equation? What is modular (clock) arithmetic, and how can we use remainders to complete a chart of	• Activity should be preceded by reading about finite differences in UMSM, pp. 168-181. Assign questions about the reading including sample problems. Groups present results, including equations they developed. Note: these differences may have been covered previously in the course if some participants were familiar with them, and used them in presenting their solutions. • Mini-tutorial as needed, to clarify key ideas regarding

		values? Will the properties of the real number system hold true for a modular arithmetic set? (always, sometimes, never)	first and second differences. • Introduce modular or clock arithmetic and assign several sets to be completed (naming a specific mod over addition or multiplication). Minimum suggested – at least one prime and one composite number. • Hand out a list of the properties of the real number system (as covered in the first algebra course) and ask groups to test these properties on their modular arithmetic sets. Discuss results? Ask questions such as "Is the system associative over addition? Does each element of your set have a multiplicative inverse? Is there any difference between the set generated by a prime or non-prime number? Why or why not?
15	Arithmetic and geometric sequences Families of functions	What is characteristic of a numerical pattern that can be designated a sequence? What is the difference between an arithmetic pattern or a geometric one?	• Introduce arithmetic and geometric sequences, emphasizing the rate of change. (see EUF, pp. 29-33). Compare recursive rules to rules in terms of n. • Assign problems related to sequences. • Have groups brainstorm families of functions and make lists of all characteristics they can name for linear, quadratic, exponential, and rational functions. Whole class discussion – come up with

			the master list.

Algebra for Middle School Specialists
Class 9: Sample lesson from one instructor

Goals for the class:

- Review exponents and their general properties.
- Explore exponential functions
- Compare exponential graphs to graphs of linear functions.
- Use graphing calculators to examine exponential functions
- Use a task from a real world context that provides an experience in using an interdisciplinary approach to teaching mathematics.
- Engage in curve fitting with linear and exponential functions to analyze, represent, and make predictions from a given data set

Debriefing Homework (20-30 minutes)

- **Ask participants to** share the implementation of any activities or strategies from the project they have used in their classrooms. What worked well, what challenges are they bumping up against. Try to identify a few folks who are beginning to push away from more traditional textbook-like approaches to using more tasks in their classrooms. How are they balancing skill development with conceptual development? (10-15 minutes)

Lesson Launch

Based on the needs of the class:

- Review exponents and their general properties.
- Explore exponential functions.
- Compare exponential graphs to graphs of linear functions.
- Use graphing calculators to examine exponential functions

New Material

Exploring data sets: Based on the August 2010 MT activity, "How Long Does it Take a Person to Sober Up? Some Mathematics and Science of DUI" to investigate curve fitting using linear and exponential functions. This is a good example of an interdisciplinary unit and also illustrates the rich materials and often like this one, classroom-ready tasks, that are published each month in the NCTM journals. Instructors should review the article before using the activities with participants. Do not provide the article to students until after the online session.

The Before: This activity is formatted in such a way that it can be used to highlight the Van de Walle Before-During-After task-based lesson framework introduced in the leadership classes.

Ask students to discuss the following and have some discussion about their ideas.

a. What happens to various drugs in our body over time?

b. What can be considered a drug?

c. Assume that a drug is ingested into the body, what happens over time?

Students complete the activity:

How Long Does It Take a Person to Sober Up?
Some Mathematics and Science of DUI

<u>Task Based Lesson: Before Stage</u>

- What are some things you notice about the table?
- By hand, sketch a quick graph to illustrate what happens to the drug in the first 12 hours after it has been ingested..

Table 1: The Case of Penicillin	
0	600,000
0.5	300,000
1.0	150,000
1.5	75,000
2.0	37,500
2.5	18,750
3.0	9,375
3.5	4,688
4.0	602,344

The graph is sketched by hand rather than using the graphing technology tools. Ask a student to share and support his/her graph and what information can be learned from this model or representation of the data. Then ask a student who has something different to share and explain how they thought differently about the graph. Based on what you know about your learners you will have other questions you want to pose.

After this discussion based on the hand sketched graphs, the instructor will then demonstrate what the graph would look like using the graphing calculator. If you have a group that has had little experience with curve fitting, you may need to spend some time demonstrating how to use the calculator to do that.

This would be a good time to discuss when asking students to complete an activity without the graphing calculator is beneficial to develop conceptual understanding. It is important to have the conversation about appropriate calculator use and when an instructional concept may emerge more clearly by exploring first without the calculator. Another good question here is to ask the group what mathematics they could bring to light with their students by using this task in class. (20-30 minutes)

The During: Working in groups of 3 or 4 <u>participants will need HO 1 (How Long Does it Take a Person to Sober Up Activity Sheet 1: Caffeine Elimination Rate and Activity Sheet 2: Alcohol Elimination Rate). (HO from the article)</u> This activity may push on the mathematics that some of the participants bring to the class. It will be helpful to have a strong math person in each group. (45-60 minutes)

Working in small groups participants will complete activity one and two and respond to the questions on the activity sheets. As they are working on the activity ask the participants to think about how this activity could be in Math 8 or Algebra I. What modifications would need to be made in the task itself or of the questions on the activity sheet for any of the courses, what prior knowledge would students need to bring to the tasks, what mathematics concepts, skills, and vocabulary can be highlighted with this activity, and what information do you as instructors need to clarify for yourself in order to use this activity in class. When the whole group comes back together be ready to discuss these questions and any of the questions on the activity sheet. Note: The last question on Activity 2 is, "Write a short reflection on this investigation, summarizing what you have learned as well as any personal reactions." You may want folks to skip this and use it for a journal prompt.

The After: Facilitators may have picked up on some mathematics that needs to be brought to light from visiting the various small groups as participants worked on the activities. (15-30 minutes)

To highlight the differences in linear functions and exponential functions a question that asks participants to make comparison statements about Caffeine Elimination Rate and Alcohol Elimination Rate and what are some health implications that one might take away from this experience.

Use the questions posed for the participants to think about in The During to have them discuss how they might use this activity in class. At this point in time, some participants may still be resistant to thinking about using tasks to teach mathematics rather than the more traditional skill driven methods they are more familiar with. Through these discussions windows may begin to open that will encourage the more resistant in the group to begin thinking about using tasks periodically in their work. Perhaps setting a goal of trying to use a rich task at least once every two weeks.

1. **Investigating Best fit Line:** Providing there is time left, use the text, *Focus in High School Mathematics: Reasoning and Sense Making Algebra.* The section, Fitting Lines to Data, can be found beginning on page 31 and ending on page 35. This may be a good time to take some time in class to read these pages together and help participants make sense of this idea.
2. **Alternative Activity:** The following activity will provide an opportunity for participants to engage in an activity to collect data and then make a generalization that will call upon their knowledge of polynomial functions.

   Task: Use text *Mathematics Assessment: A Practical Handbook for Grades 9-12.* See page 132, The Box Investigation. Use question 7 to explore the relationships of height to volume, height to length, height to area of bottom of box.

   Have the participants develop the generalization first without the calculator and then use the graphing calculator to enter the data into lists and explore the curves. Have them develop sets of questions they could ask their students to bring out mathematical ideas such as domain, range, zeros, etc.

Homework Assignment

Web Site Investigation **http://www.wmueller.com/precalculus/families/splash.html**

The site contains summary information on eight major families of functions used throughout precalculus and calculus. Each family is examined in four common representations. Browse through it, see what is here, and come back when you need to learn (or be reminded of) the idiosyncrasies of a particular family's behavior.

Geometry and Measurement for Mathematics Specialists

Geometry and Measurement is a 3-credit hour graduate mathematics course designed to prepare prospective Mathematics Specialists to coach teachers of geometry up to, but not including, high school geometry. Numbers and Operations is a prerequisite for this course. The course focuses on a deep study of the geometric ideas in the K-8 curriculum as well as on the study of how students can develop their understanding of these ideas.

Course Overview and Goals

The course uses physical and visual activities, the examination of student case studies, Geometer's Sketchpad activities, and traditional instruction to develop basic geometric and measurement ideas and concepts.

The students' conceptual development in K-8 geometry (and measurement) is centered on the conceptual structuring of space in dimensions 1, 2, and 3. The basic structures that students need to develop for understanding of this informal geometry include:

- A conceptual development associated with length, area, and volume that include a solid and flexible collection of measurement skills and the fundamental relationships connecting concepts from these domains – including formulas
- A conceptual development of angles and angle measurement – including an emphasis of angles as turns
- Lines, especially parallel and perpendicular
- The classification and analysis of shapes
- A basic understanding of symmetry and geometric transformations

Course Materials

Primary Student Materials

Schifter, D., Bastable, V., & Russell, S. (2002). *Examining features of shape: Casebook*. Parsippany, NJ: Dale Seymour Publications.

Schifter, D., Bastable, V., & Russell, S. (2002). *Measuring space in one, two, and three dimensions: Casebook*. Parsippany, NJ: Dale Seymour Publications.

Smith, M., Silver, E., & Stein, M. (2005). *Improving instruction in geometry and measurement*. New York: Teachers College Press. (IIG&M)

Geometer's Sketchpad Software (GSP)

Primary Instructor Materials

Bastable, V., Schifter, D., & Russell, S. (2002). *Examining features of shape: Facilitator's guide*. Parsippany, NJ: Dale Seymour Publications. (DVD)

Schifter, D., Bastable, V., & Russell, S. (2002*). Measuring space in one, two, and three dimensions: Casebook.* Parsippany, NJ: Dale Seymour Publications. (DVD)

Additional Instructor Materials

Battista, M. (2009). Highlights of Research on Learning School Geometry. In T.V. Craine (Ed.), *Understanding Geometry for a Changing World: Seventy-first Yearbook* (pp. 91-108*).* Reston, VA: National Council of Teachers of Mathematics.

Berkas, N., & Pattison, C. (2007). *Explorations with the XY coordinate pegboard.* Vernon Hills, IL: ETA/Cuisenaire.

Clements, D. (2003). *Classroom activities for learning and teaching measurement*. Reston, VA: National Council of Teachers of Mathematics.

Clements, D. (2003). *Learning and teaching measurement.* Reston, VA: National Council of Teachers of Mathematics.

Erickson, S., & Thiessen, R. (2003). *Looking at geometry*. Fresno, CA: AIMS Education Foundation.

Fears, C. (n.d.). Puzzling pentominoes. Retrieved June 7, 2015, from http://illuminations.nctm.org/Lesson.aspx?id=4105

Gillespie, N., & Buell, J. (1995). *Mira activities for the middle grades*. Canada: Mira Math.

Interactives 3D shapes: An introduction. (n.d.). Retrieved June 7, 2015, from http://www.learner.org/interactives/geometry/index.html.

Johnston, C. (n.d.). discovering the area formula for circles. Retrieved June 7, 2015, from http://illuminations.nctm.org/Lesson.aspx?id=1852

Lawrence, A., & Hennessy, C. (n.d.). orange you glad. Retrieved June 7, 2015, from http://mathsolutions.com/documents/9780941355810_L2.pdf

Lund, C. (1980). Dot paper geometry: *With or without a geoboard.* New Rochelle, NY: Cuisenaire Company of America.

Mira math activities for elementary school: A new dimension of motivation and understanding. (1973). Willowdale, Ont.: Mira Math.

Pocket protractor: Measure angles with a tool you can make yourself. (n.d.). Retrieved June 7, 2015, from http://www.exploratorium.edu/geometryplayground/

Activities/GP_OutdoorActivities/PocketProtractor.pdf

Russell, R. (n.d.). Rediscovering the Patterns in Pick's Theorem. Retrieved June 7, 2015, from http://illuminations.nctm.org/lesson.aspx?id=2083

Serra, M. (1994). *Patty paper geometry.* Berkeley, Calif.: Key Curriculum Press.

Course Assignments

There are numerous assignments and activities. Each is directed toward the goals of the course.

Student Case Studies

A major component of the course is reading case studies of how students confront important mathematical ideas. Typically, the teachers read the case studies for homework, discuss them in small groups using focus questions, and then discuss them in a full-class format. Teachers are regularly asked to come to the front of the class to share their ideas using newspaper print.

Geometer's Sketchpad Activities

Typically, the Geometer's Sketchpad activities were on mathematics that was also being considered for the study of student case studies.

Geometer's Sketchpad is used much more frequently in middle school than in grades K-5. Therefore, in some classes with no or few prospective middle school teachers, the Geometer's Sketchpad was not utilized.

Exit Cards

At the end of most classes, teachers are asked to respond to questions on an Exit Card. This activity helps the teachers focus on key ideas and is one more tool to help the instructors learn what each teacher in the class is thinking.

Mathematics Papers

Two mathematics reasoning papers are typically assigned. These papers are designed so that the teachers are required to write about mathematics. The teachers are not expected to discover the mathematics, but rather to write about mathematics that has been explored in the class and/or described by an instructor. The first paper is assigned on day 3 and is described in detail as a part of the description of that day's activities.

The second paper concerns the Koch Snowflake. It involves ideas from three of the Mathematics Specialists courses: *Geometry and Measurement*, *Rational Numbers and Proportional Reasoning*, and *Algebra and Functions I*. In class, we develop and discuss the necessary tools to analyze the triangle, including the formula for the sum of the terms of a geometric sequence and the comparative area of a triangle whose linear dimensions are ⅓ that of the original triangle. The following is a part of the instructions provided:

Creation of the Koch Snowflake
Start with an equilateral triangle with each side being of length 1.

Then follow this iterative process:

1. Divide each line segment into three segments of equal length.
2. For each line segment, draw an equilateral triangle that has the middle segment from step 1 as its base.
3. Remove the line segment that is the base of the triangle from step *2*.

The figure obtained by following this process once is called *stage 1*.

When we follow the iterative process on *stage 1*, the result is called *stage 2*.

When we have followed the iterative process *n* times, we call the resulting geometric figure *stage n*.

When the process is followed an infinite number of times, we call the resulting geometric figure the *Koch Snowflake*.

Body of Paper
The body of the paper should include a description of the geometric figure that is obtained by following this iterative process as well as careful drawings of at least the first three stages. Then it should contain formulas that give:

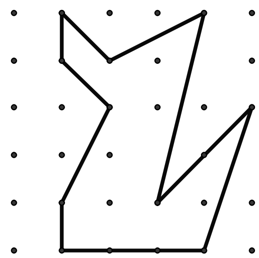
- The perimeter of the figure obtained at *stage n*.
- The sum of the areas of all of the new triangles added to the figure at *stage n*.
- The total area of the figure after *n* stages.
- The perimeter of the figure after an infinite number of stages.
- The total area of the figure after an infinite number of stages.

Teachers are provided with help to develop these formulas and are asked to explain this development in Appendices. The resulting figure has finite area and infinite perimeter.

<u>Midterm and Final Test</u>
A Midterm and Final test are given. These tests are viewed as necessary since, as compared to a continued education workshop, this course carries graduate credit and, completion of the overall program provides certification that the teacher is prepared to carry out a new responsibility, that of a Mathematics Specialist. Prior to each, a practice test is given to the teachers, with an opportunity to obtain help from the instructors if needed. The Final test included the following questions:

1.

a. Find the area inside the shape below using the chop method. Explain in detail.

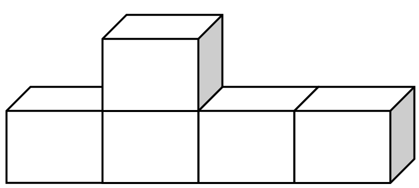


b. Find the area using Pick's Theorem. Show your work.

2. What is the width of a rectangle of length 12 inches if it is similar to a rectangle with length 8 inches and width 6 inches?

3. Complete the table for the following figure.

SCALE FACTOR	SURFACE AREA	VOLUME
1		
2		
3		
4		
. . n		



4. Find the height of a right circular cone that has a volume of 200 inches^3 and has a base of radius 10 inches.

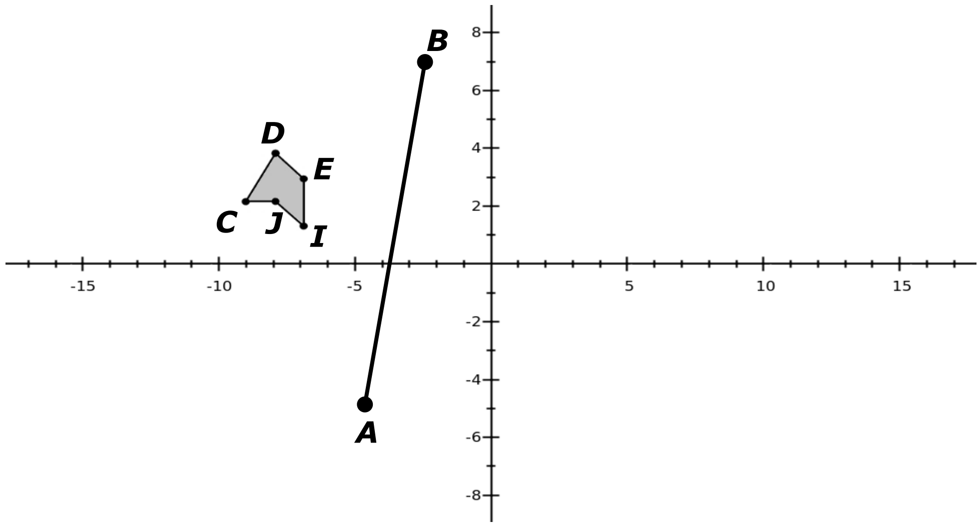
5. What is the formula for the volume of a pyramid with a rectangular base of sides *L* and *W* height of *H*?

6. What is the formula for the area of a circle with radius *r*? Explain how we demonstrate this formula is plausible.

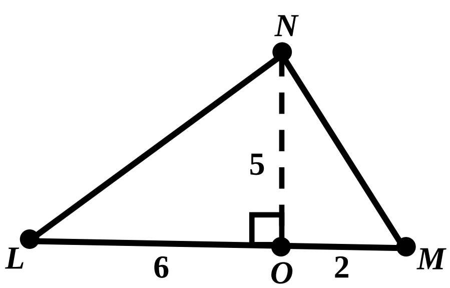
7. Show that the sums of the first *n* terms of the geometric sequence $a, ar, ar^2, \ldots, ar^{n-1}$ are given by the formula $S = a\dfrac{(1-r^n)}{(1-r)}$.

8. Reflect the polygon below across the line *BA*. What can you say mathematically about the distance from the vertex of a pre-image to the corresponding vertex of the image?

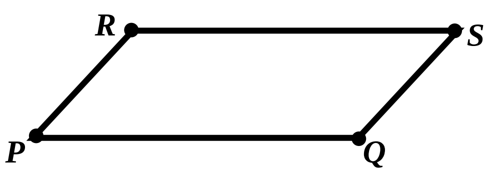
What can you say mathematically about the angles formed by this reflection? Label the image.



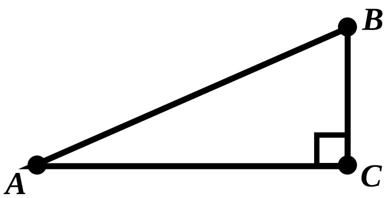
9. Using only the formula for the area of a rectangle, tell how you could determine the area of the following figure by cutting and pasting.



10. Using only the formula for the area of a rectangle demonstrate how to find the formula for the area of a parallelogram by cutting and pasting.



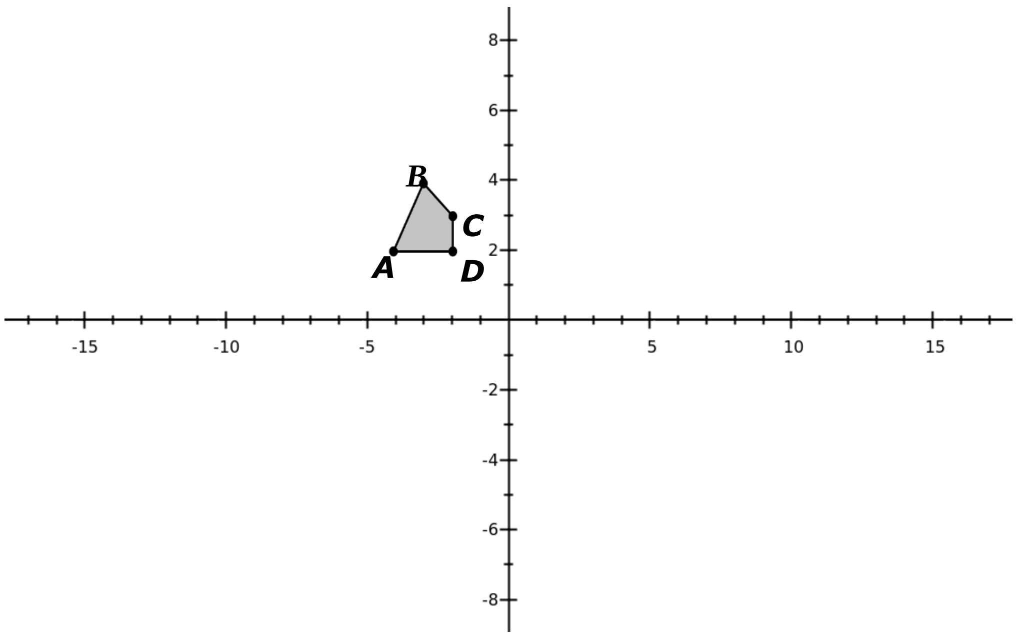
11. Given a segment *AB* of length *C* and the right triangle below, show that $A^2 + B^2 = C^2$. Use the area without dot paper worksheet.



12. Complete the following table for *two-dimensions* with at least five entries for each column.

WHAT IS MEASURED	TOOLS	UNITS

13. Translate the figure -5 units vertically and +6 units horizontally. Label translated the image.



14. Construct a Magic Rhombus Garden using Geometer's Sketchpad.
    a. Show grid, use snap points as needed.
    b. Construct a rhombus, using a circle with a radius of 3 units.
    c. Sketchpad History can be used to validate the method of construction.
    d. Create a parallelogram with a height of 12 units and a width of 14 units (measure the lengths of the sides and drag to appropriate sides).
    e. Measure the sides of a rhombus and drag rhombus to the upper left portion of the parallelogram interior.
    f. Divide the garden into two equal regions with one line, including both quadrilaterals. Make use of your knowledge of parallelograms to show the division is accurate.
    g. Show your two area measures to verify your conclusions. Describe your moves in a text box with your first initial and last name on the drawing and save in a doc file on a flash drive.

Final Reflection Paper
In their third major paper, the teachers are asked to reflect on how what they learned in the course will impact their teaching.

The work of the teachers is assessed, and immediate feedback is provided. Course grades are determined as follows:

Course Outline: Topics and Essential Questions
The overview displayed in Figure 6 presents the scope and sequence of topics in the Geometry and Measurement **course taught in a 2-week summer institute**. Each class is approximately 7 hours unless otherwise noted. The overview identifies the topics and essential questions for each class and resources used to support each class. The course textbooks are noted in the outline as follows.

MS123: *Measuring Space in 1, 2, and 3*
IIG&M : *Improving instruction in geometry and measurement*.
GS: Geometry Sketchpad

Figure 7. Geometry and Measurement Course: Overview of Topics.

Class	**Topics/Resources/Activities**	**Essential Questions**
1	Van Hiele Discussion (Battista) Ordering Rectangles Activity (MS123) Crazy Cakes Activity (MS123) Perimeter and Measuring Perimeter Activity (MS123) **EXIT CARD:** Think about the activities completed today. Identify the Van Hiele level of one of the activities. The level you discuss may reflect your level or the level a student must possess to access the activity. **HOMEWORK** • MS123 Chapter 1 Cases • "The Case of Isabelle Olsen" (IIG&M)	What are the goals of the course? What geometrical/ mathematical ideas are needed before individuals can address more advanced ideas? How can geometric shapes be sorted, both visually and analytically? What do we mean by 'perimeter'? What makes good mathematical definitions?
2	Area and Perimeter Activities (Clements) Area and Perimeter with Geoboards (Lund) Discuss "The Case of Isabelle Olsen" (IIG&M) and	What is the relationship between area and perimeter? How does this relationship

	MS123 Chapter 1 Focus Questions Magic Gardens Activity (MS123) GSP Demo Activity Participants create a GSP Magic Garden **EXIT CARD:** How has today's lesson changed your ideas on the teaching of perimeter and area? (front) How was today's session for you as a learner? (back) **HOMEWORK** • Opening activity for "The Case of Barbara Crafton" (IIG&M) • Read "The Case of Barbara Crafton" (IIG&M) • Cases 18, 19, and 22-24, Area of triangles (MS123)	change when constraints are added or removed? (e.g. What if the geometric figure is required to be a square?) What similarities and differences occur between how K-8 students and teachers approach mathematical ideas?
3	Discuss cases from homework involving area of triangles (grades 3 & 4) and changing units (middle school) Class discussion of linear measurement: What is measured, tools, and units Assign Mathematical Reasoning paper (as described in sample lesson plan for Class 3) Geoboard Activity: Pick's Theorem (Russell) Pythagorean Theorem activities (Erickson) **EXIT ACTIVITY:** Rope Challenge **EXIT CARD:** Summarize your thinking on the introduction to your Mathematical Reasoning paper. **HOMEWORK** • Pythagorean Word Problems (Erickson) • Begin Mathematical Reasoning paper	What are the standard formulas for the area of two-dimensional figures? Which formulas are intuitively true? How can the truth of some formulas be demonstrated formally? What is the Pythagorean Theorem? Why is it valid? How can it be used?
4 (3½ hours)	Small group discussion of Pythagorean Word Problems, then shares with entire class Full class discussion of Mathematical Reasoning	What are applications of the Pythagorean Theorem? How can surface area of

	paper Discovering formula for the area of a circle (Johnston) Pentominoes/Introduction to Nets (Fears) Discovering surface area of rectangular prisms and cylinders (Interactive) **EXIT ACTIVITY:** Geometer's Sketchpad Activity **HOMEWORK** • Opening activity for "The Case of Keith Campbell." (IIG&M) • Read "The Case of Keith Campbell" (IIG&M) • Complete Mathematical Reasoning paper	rectangular prisms and cylinders be determined?
5	Collect Mathematical Reasoning paper Discovering formulas for surface area of pyramids and cones (Interactive) Class discussion of two-dimensional measurement: What is measured, tools, and units Assign Koch Snowflake paper as described in Key Activities Cut up the skin of an orange to make the formula for the surface area of a sphere appear plausible (Lawrence) Review formulas for volume of figures and then confirm using Power Solids (Interactive) **EXIT ACTIVITY:** Work on midterm practice test **EXIT CARD:** For what topics are you on shaky ground, given the midterm practice test? **HOMEWORK** • Study for midterm test • Complete practice midterm test • Begin to think about Koch Snowflake paper	What are the formulas that can be used to determine the surface area of rectangular prisms, cylinders, pyramids, cones, and spheres? What are the formulas that can be used to determine the volume of a certain figure? How far can we push the possible relationship between perimeter and area?

6	Return Mathematical Reasoning paper and discuss participant's work Discuss "The Case of Keith Campbell" (IIG&M) in table groups and entire class Participants create different size open top boxes by cutting squares of size *x* from corners of an 8.5 by 11-inch piece of paper; Participants put the volume of the box vs. *x* on the overhead and later graph $v=(x)(8.5-2x)(11-2x)$ (MS123) Participants read and then discuss "The Case of Barbara Crafton" (IIG&M) Work on Pentominoes (Fears) Midterm test **EXIT CARD:** For the past few days, we have participated in many activities measuring objects in 2D and 3D. Describe a lesson you would use to help teach these concepts. (front) If we had a chance to look at one thing AGAIN – what would you want it to be? (back) **HOMEWORK** • Paper Box Activity • Chapter 6: FQ1 – Making Paper Boxes (MS123) • Chapter 8 RESEARCH (MS123)	What are the differences among memorizing formulas, understanding why formulas are valid, and formal proof? How can we see the volume of a figure? What is the effect on the volume when linear attributes change? Can you see this visually? How is this reflected in formulas?
7	Return Midterm Exam and discuss Table and group discussion MS123 Chapter 6 Focus Questions Class discussion of three-dimensional measurement: What is measured, tools, and units MS123 Chapter 8 RESEARCH discussion using jigsaw approach Discuss highlights of RESEARCH from MS123	How do students begin to develop concepts of area and volume? How do we think of angles? As fixed space? As motion? How can the truth of some formulas be demonstrated formally? How can Geometer's

	Chapter 8 Angle Exploration: Concept of Motion/Turns Making protractors (Pocket) **EXIT ACTIVITY:** Geometer's Sketchpad Activity: • Complementary/Supplementary Angles • Vertical/Adjacent Angles • Parallel Lines Cut by a Transversal • Alternate Interior Angles • Alternate Exterior Angles • Corresponding Angles **HOMEWORK** • EFS Chapter 3 Cases	Sketchpad help us understand angles?
8	Discuss tools needed for the Koch Snowflake paper Table and group discussion EFS Chapter 3 Focus Questions Transformations: Rotations Transformations: Dilations Patty Paper and Coordinate Plane Pegboards (Serra and Berkas) **EXIT ACTIVITY:** Geometer's Sketchpad Activity: Rotations and Dilations **HOMEWORK** • Complete Koch Snowflake paper • Read EFS Chapter 4	How do we think about angles? What does it mean for a triangle to have $180°$? Or a circle $360°$? How many different ways can we visualize and experience rotations and dilations?
9	Collect Koch Snowflake paper Discussion of observation versus proof Transformations: Reflections Patty Paper/Coordinate Plane Pegboards (Serra and	Again, what are the characteristics of a good mathematical definition? What is known about how students develop an

	Berkas) MIRA Activities (MIRA and Gillespie) Transformations: Translations – Patty Paper and Coordinate Plane Pegboard (Serra and Berkas) Geometer's Sketchpad Activity: Reflections and Translations EFS Chapter 4 discussion on creating and applying definitions Read EFS Chapter 8 EFS Chapter 8 research discussion using jigsaw approach **EXIT ACTIVITY:** Practice final test **HOMEWORK** • Complete Final Reflection paper • Study for final test	understanding of shape?
10 (5 hours)	Collect Final Reflection paper Discuss what we learned doing the Koch Snowflake Creating solid figures using newspaper rods Transformations: Reflections Patty Paper and Coordinate Plane Pegboards (Serra and Berkas) Answer participant questions on final test Final test	How can we experience volume?

Geometry and Measurement
Sample Lesson Plan for Class 3

<u>Essential Questions</u>

- What are the standard formulas for the area of two-dimensional figures?
- Which formulas are intuitively true?
- How can the truth of some formulas be demonstrated formally?
- What is the Pythagorean Theorem?

- Why is it valid? How can it be used?

Welcome and Logistics (15 minutes)

- Before teachers arrive, nameplates placed on each table (six tables with four teachers at each table)
- Discuss Today's Agenda. State which instructor will lead each activity (the other instructor will participate, usually sitting at one of the tables)
- Instructors' report on Exit Card entries
- Short whole-class discussion on Magic Garden activity
  - Was it effective to consider how students encountered the activity, then doing it yourself using paper and pencil and then doing it by making use of Geometer's Sketchpad?

Case Studies (2.25 hours)

- Table discussion of "The Case of Barbara Crafton" (IIG&M): Reasoning about Units for Linear and Area Measurement. Make use of "Analyzing the Case" suggestions on page 18. (30 minutes)
- Table discussion of Cases 18, 19, and 22-24 (MS123). (1 hour)
  "From rectangles to triangles and trapezoids" – Consider the following questions from the MS123 Study Guide:
  - In Janine's case 18, line 141, Janine remarks that perimeter might seem easy, but, considering her students' discussion (lines 57-97), she sees they are still confused. What is the source of their confusion? What units are used to measure perimeter?
  - In Georgia's case 19, what differences do you notice among the way the first three children draw their arrays? Kalil writes, "I don't really get this." What doesn't he get? Follow Kalil's thinking through day 1 and day 2. What does Kalil figure out as he works with the teacher? What does his final diagram show about his current understanding?
  - **Area of parallelograms**
    - Consider parallelograms such as those drawn below. Determine a way to find the area of each. Devise a general rule for the area of a parallelogram and explain how you know it will always work.

  - **Area of trapezoids**

   - Consider several trapezoids and find the area of each. Devise a general rule for the area of a trapezoid and explain how you know it will always work. You may refer to Sandra's case if that is helpful.

- During this time, the instructors rotate from table to table, participating in the discussions. With about 15 minutes remaining, each table is assigned a question from either "The Case of Barbara Crafton" or the MS123 questions. They are to prepare a five-minute presentation, addressing their question.
- Presentations by each table, followed by a full-class discussion. (45 minutes)

<u>Full-Class Discussion of Linear Measurement (15 minutes)</u>
The discussion begins by considering the following table:

WHAT IS MEASURED	TOOLS	UNITS
Distance travelled Circumference of a circle Distance from the earth to the sun	Ruler Measuring wheel Chain	Inches Miles Light years

Typically teachers come up with more than 10 entries for each column.
<u>Assign Mathematical Reasoning Paper (15 minutes)</u>
The following paper was assigned. It was created using the ideas in the MS123 Facilitator's Guide. It will be due in two days.

PAPER I
We have been thinking about and using the formulas that can be used to find the area of various shapes. In this short paper, your task is to explain in clear English why a couple of these formulas are valid.

I. <u>Triangles</u>

For any vertex *B* in a triangle, we can consider the side opposite *B* to be the base of the triangle with length *b*. We can call the length of the perpendicular line between *B* and (possibly the extension of) the base the height, *h*, of the triangle. We want to demonstrate that for any triangle with any vertex that the area of the triangle can be determined by using the formula $A = \frac{1}{2}bh$.

Carefully explain why the formula works for the following special cases:

- A right triangle where the side opposite vertex *B* is a leg (not the hypotenuse) of a right triangle.

- An isosceles triangle where the two congruent sides meet at vertex *B*.
- A triangle where the perpendicular line from *B* intersects the base.
- A triangle where the perpendicular line from *B* does not intersect the base. (The method you used to demonstrate the previous case will probably not work. If this is so, you may show why it doesn't work and skip this case for now.)

II. Parallelogram

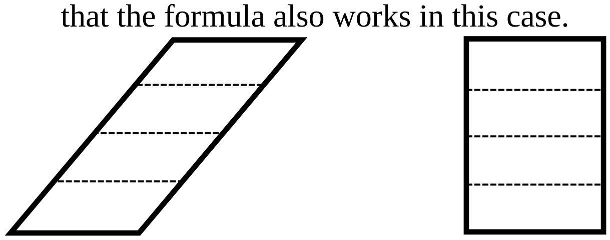
For any vertex *B* in a parallelogram, we can call the height, *h*, of the parallelogram the length of the perpendicular line from *B* to an edge (or possible extension of an edge) opposite *B*. We can call the edge opposite *B* the base of the parallelogram, with length *b*. We want to demonstrate that for any parallelogram, the area of the parallelogram can be given by the formula $A = bh$.

First: Explain carefully why this formula works in the following special case:

- A parallelogram where the perpendicular line from *B* intersects the base.

Next: Consider the other case where the perpendicular line from *B* does not intersect the base. Does your method work? If not, show why.

If your method does not work, you may wish to consider the following decomposition of a parallelogram. It should help you figure out an argument to show that the formula also works in this case.



III. Triangles revisited

Now, return to the final case of a triangle. If you skipped this case, carefully finish the argument now. One approach would be to create a parallelogram whose area is twice the area of your triangle.

You may talk to anyone in the class or anybody else and look in any notes. Case Studies 22-24 in Measuring Space in One, Two and Three Dimensions should be useful. Each person should turn in your own paper, and it should be in your own words. You should type up the paper, but any diagrams, figures, etc. may be written on your paper. Your audience should be a sixth-grade teacher who likes mathematics, but who has not thought about why the area formulas are valid. You can assume that the teacher knows the area of any rectangle with length L and width W is given by $A = L \bullet W$. **Be sure to cite any references that you are using other than the *Developing Mathematical Ideas* materials.**

GRADING RUBRIC

Addressed all parts of the assignment in a thoughtful manner, including diagrams	35%
Good description in Part I that accurately describes each case	15%
Good description in Part II that accurately describes each case	15%
Accurate description in part III, including diagrams	15%
Correct grammar, spelling; well-written paper	15%
Something WOW!!!	5%

<u>Geoboard Activities: Preliminary explorations (45 minutes)</u>

Each teacher is provided with a geoboard and rubber bands. They are asked to find the area of various regions. After finding areas with no structure, one teacher describes to the whole class how to decompose a region into triangles and rectangles to find the area of the entire region. Another teacher shows how to view the region as a part of something bigger whose area is easy to determine and then to calculate the area of the parts that need to be chopped off. This is called the chop method.

<u>Lunch Break (1.25 hours)</u>

<u>Geoboard Activities: Pick's Theorem (1.5 hours)</u>

Continue working with the chop method. Then consider Pick's Theorem, which states that "If the vertices of polygons are lattice points, then the area, *A*, can be found using the formula $A = \frac{b}{2} - 1 + i$, where *b* is the number of lattice points on the boundary, and *i* is the number of lattice points in the interior." There are a number of ways to have the teachers discover/confirm this result. One way to begin is to determine the formula for *A* if there are *b* lattice points on the boundary and no lattice points in the interior. The teachers are encouraged to work together at their tables.

<u>Pythagorean Theorem Activities (45 minutes)</u>

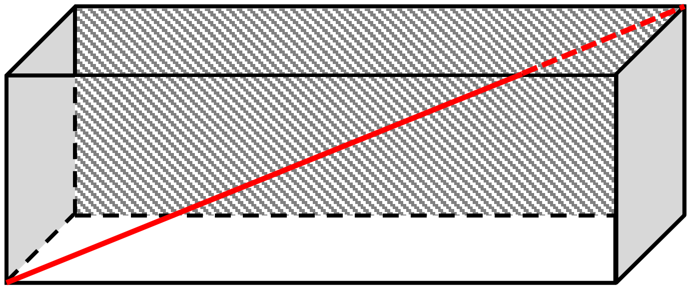
First, the teachers are asked to create a 3x4 right triangle on their geoboards and then compute the areas of the squares created off of each of the sides of the triangle. Then some other ways of demonstrating the Pythagorean Theorem are presented.

<u>Exit Activity, Exit Card, and Assignment of Homework (45 minutes)</u>

Forty-five minutes from the end of the class, the teachers are given their homework for the next class and assigned their Exit Activity and Exit Card. They may leave after they complete their Exit Activity and Exit Card. Many remain after that to begin their homework.

Exit Activity:

Your instructor wants to string a rope from the ceiling in one corner of the classroom to a corner on the floor on the opposite side. How long will the rope have to be? Provide an explanation detailing the steps you use to complete the task. Then provide your response showing your work.



Exit Card:

Summarize your thinking about the introduction to your Mathematical Reasoning paper.

Homework:

- Standard problems requiring the use of the Pythagorean Theorem
- Begin Mathematical Reasoning paper

Probability and Statistics for Mathematics Specialist

Probability and statistics for Mathematics Specialist is a 3-credit hour graduate mathematics course designed to prepare teachers with at least 3-years of classroom teaching experience to become school-based mathematics specialists. The course will develop a conceptual understanding for teaching probability and statistics topics in the K-8 curriculum. Topics include but are not limited to basic rules of probability, including dependence and independence; graphical representations of data; notions of center and dispersion of data; common misconceptions; the central limit theorem; selected topics in linear regression. Special attention will also be given to children's thinking, how they learn mathematics, their problem-solving strategies, and how they construct their understanding of key mathematics concepts.

Course Goals

This course is designed to engage participants in constructing relational understanding in the following areas:

- theoretical development of mathematics and students' learning of mathematics within the content strands of data analysis, statistics, and probability; and
- development of pedagogical content knowledge of data analysis, statistics, and probability appropriate for K-8 Mathematics Teacher Specialists.

The course will enable participants to

- Collect and analyze data.
- Use probability to predict outcomes.
- Develop experiments.
- Understand sampling distributions.
- Understand the relationship between theoretical and experimental probability.
- Demonstrate an understanding of ways in which children learn mathematics.
- Experience the process of formulating an interesting question and gathering data to answer it.

Course Overview

Class sessions will engage participants in the hands-on inquiry of necessary to support effectively teaching K-8 statistics and probability topics that prepare students for further study in statistics and probability. Small groups and pairs engage in problem solving and case discussions, whole-group presentations, and integration of technology when appropriate. Frequently class time is spent to generate data to illustrate a concept or for use in data analysis. Whole-class activities and discussions that model effective classroom practices that are advocated in the *Virginia Standards of Learning* (VDOE, 2009*)*, the *Professional Standards for Teaching Mathematics* (NCTM, 1991), the *Assessment Standards for School Mathematics* (NCTM, 1995), and the *Principles and Standards for School Mathematics* (NCTM, 2000).

During the course students develop a deep and connected understanding of mathematics content knowledge and a flexible set of proven effective pedagogical skills through the following activities.

1. Examine the specific conceptual knowledge and procedural knowledge related to data analysis, statistics, and probability;
2. Interpret and assess K-8 students’ written work related to data analysis and statistics portrayed in the Developing Mathematical Ideas (DMI) materials of written and video cases;
3. Participate in a 4-step process of conducting a statistical investigation and share results;
4. Examine mathematics curriculum materials to determine which mathematical ideas it could raise for students for specific lessons and how a teacher may implement those ideas; and
5. Explore how to select and implement the use of physical manipulatives and computer websites in K-8 classrooms.

The course has been taught in a variety of formats. In two-week residential summer institutes with 54 hours of class time and significant daily in class work and homework assignments, in a regular semester course with 15 three-hour sessions. The timeframe when the course is offered impacts participants experience in different ways. In the summer, students immerse themselves in the work and have the opportunity for additional collaboration with their peers after class hours. However, in the summer there are few if any relevant opportunities to transfer the coursework to their own classrooms.

The instructional methodology assumes a student-centered, inquiry model and makes use of small groups and whole group discussions based on mathematics tasks, written and video cases, and cooperative group work around mathematics content and mathematics content pedagogy. Doing mathematics together is an important part of the course. Mathematics task and activities are included in the course text and instructors supplement with additional tasks based on students' needs. In addition, when the course format allows, the students transfer their learning from the course to their own classroom practice by analyzing their own students' work samples, interviewing individual students, and writing cases from their classroom practice. Course instructors bring attention to the mathematics content, the developmental trajectory of the mathematical concepts, how children make sense of the mathematics, and which pedagogical moves afford students opportunities to become mathematical thinkers.

While mathematics is the focus of the course, it is also important to include the textbook video and case studies as a context to deepen students' understanding of how K-8 learners make sense of the mathematics. The case studies offer validity to the learner-centered way of teaching and making sense mathematics. In addition, the cases illustrate the critical role the teacher plays in selecting tasks and orchestrating classroom discussions. Students, with instructor guidance, engage in robust conversations about the mathematics in the cases and, as a result, attain a deeper understanding of the mathematics. Students develop their mathematical understanding in such a way to as to relate the mathematics back school mathematics. There are occasions when the instructor needs explicitly to connect the case study discussion with the mathematics discussions or to supplement with activities that encourage additional amplification of the mathematics.

Instructors and students engage in ongoing formative assessment, and as students construct their own understanding of the mathematics, and they make connections throughout the course that deepen their understanding of the mathematics content. Specific writing prompts are provided by the instructor throughout the course for the instructor and the student to evaluate the student's evolving understanding. Two summative assessments are administered, a midterm assessment and a cumulative final exam.

Course Materials

Listed below are the primary student and instructor texts for the course. In addition, instructors will include supplementary readings such as NCTM journal articles.

Primary Student Texts

Russell, S.J., Schifter, D., & Bastable, V. (2002). Developing mathematical ideas: Working with data casebook. Parsippany, NJ: Pearson Education, Inc.

Van de Walle, J.A., Karp, K.S., & Bay-Williams, J.M. (2013). *Elementary and middle school mathematics: Teaching developmentally (8th Edition).* Upper Saddle River, NJ: Pearson Education, Inc. (A student textbook for Leadership I).

Virginia Department of Education (VDOE). (2004). *Probability and statistics professional development module for elementary/middle school teachers, Revised.* Richmond, VA: Virginiaa Department of Education. Retrieved from http://doe.virginia.gov/testing/sol/standards_docs/mathematics/2001/resources/elementary/probability_module/mprobstatentire.pdf

Instructor Primary Resources

Russell, S.J., Schifter, D., & Bastable, V. (2002). Developing mathematical ideas: Working with data facilitator's guide and video. Parsippany, NJ: Pearson Education, Inc.

Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., & Scheaffer, R. (2007). Guidelines for assessment and instruction in statistics education (GAISE) report. *Alexandría: American Statistical Association*. Retrieved from http://www.amstat.org/education/gaise/GAISEPreK-12_Full.pdf

Finally, activities and articles were also adapted from the Navigating series published by NCTM. We recommend that instructors also have copies of each of these:

1. Navigating through Data Analysis and Probability in Pre-kindergarten – Grade 2
2. Navigating through Data Analysis and Probability in Grades 3 - 5
3. Navigating through Probability in Grades 6-8
4. Navigating through Data Analysis in Grades 6-8

Supplementary Student Readings

Probability Articles

Aspinwall, L. & Shaw, K. (2000). Enriching students' mathematical intuitions with probability games and tree diagrams. *Mathematics Teaching in the Middle School, 6*(4), 214-220.

Baker, J. (2013). Delving deeper: Rolling the dice. *Mathematics Teacher, 106*(7), 551-556.

Bush, S. & Karp, K. (2012). Hunger games: What are the chances? *Mathematics Teaching in the Middle School, 17*(7), 426-435.

Dowd, D. (2013). Quick reads: A circle model for multiplying probabilities. *Mathematics Teaching in the Middle School, 18*(8), 464-466.

Kimani, P., Gibbs, R. & Anderson, M. (2013). Restoring order to permutations and combinations. *Mathematics Teaching in the Middle School, 18*(7), 430-438.

Leah McCoy, L., Buckner, S., & Munley, J. (2007). Probability games from diverse cultures. *Mathematics Teaching in the Middle School, 12*(7), 394-402.

Naresh, N. & Royce, B. (2013). Dropping in on the math of Plinko. *Mathematics Teaching in the Middle School, 19*(4), 214-221.

Quinn, R. (2001). Using attribute blocks to develop a conceptual understanding of probability. *Mathematics Teaching in the Middle School, 6*(5), 290-294.

Rubel, L. (2006). Good things always come in threes: Three cards, three prisoners, three doors. *Mathematics Teacher, 99*(6), 401-405.

Williams, N. & Bruels, C. (2011). Mathematical Exploration: Target geometry and probability using a dartboard. *Mathematics Teaching in the Middle School, 16*(6), 375-383.

Course Topics and Essential Questions
The outline of class meetings that follows is based on fifteen 3-hour class meetings.

DMI contains readings and activities to help the students become comfortable with student thinking and reasoning abilities. The DMI Facilitator's Guide contains a suggested schedule for each session. It is left to the instructor to choose which DMI material from these sections they would prefer. DMI does not address probability; it is left to the instructor to identify journal articles in place of DMI materials.

Access to a graphing calculator and access to a computer lab is also recommended, in particular if students are future middle school mathematics specialist. It is up to the instructor to determine what level is appropriate based on the class make-up and students' previous experiences.

Key for course textbook references:

- DMIWWD: *Developing Mathematical Ideas: Working with Data*
- EMSM: *Elementary and Middle School Mathematics: Teaching Developmentally*
- VDOE-PS: *Probability and Statistics for Elementary and Middle School Teachers: A Staff Development Training Program*—Retrieve from http://doe.virginia.gov/testing/sol/standards_docs/mathematics/2001/resources/elementary/probability_module/mprobstatentire.pdf

Figure 8. Probability and Statistics Course: Overview of Topics

Class	**Topics/Resources**	**Essential Questions**
1	KWL: "What is important for students to learn about data and statistics?" Activity: The Big Ideas of Statistical Investigation (VDOE-PS, page 5 and page 19). Introduce the idea of a measurement center. The measurement center is used to collect data for class activities and for group projects. Every class session, post directions and have students report some form of data. Classification Attributes Object graphs Data analysis Numerical data Categorical data DMIWWD Chapter 1 Math Activity and case discussion EMSM, page 434-436	What does it mean to do statistics? What is the purpose and process of statistical investigation? What are the big ideas of a statistical investigation? What is the purpose and how does one formulate a question? What are defining characteristics of numerical (quantitative) data and categorical (qualitative) data and how do the characteristics impact the ways the data can be displayed?
Class 2	Collect and organize categorical data; Use data collected from participants to answer questions about the class. Investigate how different organization of data gives different views of the same data.	When developing a statistical investigation, what is the relationship between the question and collecting the data? When working with categorical data what considerations must be

	Line plots Frequency Predictions Categorical vs. numerical data Attributes Activity: Yekttis – Guess MY Rule from *Used Numbers*, Sorting Groups and Graphs, Dale Seymour 1990 The Process of Statistical Investigation, overview of the project so that class can begin preparing. This is a long range project and this introduction is the first step. DMIWWD Chapter 2 and 3	given to determining the categories and representing the data? How does the organization and display of some data set effect interpretations? What are the criteria for developing a good survey?
Class 3	Activity: Participants create a line plot with post-it notes that display the number of years of teaching service per person. Include a discussion of skew, range, and similar topics. Activity: Grab a Handful (VDOE-PS: page 50) Recognize outlier data Calculate the mean, median, mode, quartiles for a set of numerical data Use the appropriate measures to construct a box and whisker plot Use the appropriate measures to construct a box and whisker plot Statistics a problem-solving process (GAISE Report: http://www.amstat.org/education/gaise/GAISEPreK-12_Full.pdf) EMSM: pages 446-450 DMIWWD Chapter 7	How is describing a graph different from or similar to analyzing data? How do measures of central tendency support analyzing data and what can be learned about a population from each measure of central tendency? What can be learned from examing the spread in a data set from a particular population and within the quartiles of a data set? What are ways to What defines statistics as problem-solving process? What does it mean to be average?
Class 4	Activity: What is a Median? &	What are the characteristics that

	Class Height Data Activity: How many pockets? Individually, construct a line plot and a bar graph, based on class data, of the data. Write one sentence for each graph describing what you notice about the data. Match the Data Sets (Idea from *Good Questions for Math Teaching*, 5-8, Schuster and Anderson, Math Solutions, 2005) See the appendix, pages 88-100, in NCTM' s *Navigating through Data Analysis in Grades 6-8* to helps students begin thinking about the data project. DMIWWD, Chapter 4	differentiate categorical data from numerical data and how each can be used to draw inferences about the population the data came from? What should also be considered when using the median of a data set to characterize the population? What characteristics enable informative descriptions and comparisons of data sets?
Class 5	Activity Mystery Data: Goal is not to find the exact actual group, but rather a reasonable idea for a group of living things that might fit the data. May want to stretch this over two classes to provide reflection time before starting final conclusions. (*Used Numbers, Statistics: The Shape of the Data*, Dale Seymour 1989) Mystery Data Activity: http://www.doe.virginia.gov/testing/solsearch/sol/math/5/mess_5-15.pdf Stem and Leaf display (use height data from class 4) Finalize statistical investigation project plan with partner/s. Include any questions or areas of concern. DMIMMD, Chapter 5	What does it mean to summarize data? What characteristics enable informative descriptions and comparisons of data sets? What conclusions can be drawn from a population using a sample data set from the population and what considerations must be taken to increase the confidence in the conclusions? For a given situation, what factors impact which measure of central tendency is most applicable?
Class 6	Use activities to explore the two	Why is it important to consider

	interpretations of mean: leveling and balance point Activity: What is the mean length of our feet in inches? (EMSM, page 447) Activity to investigate variability. Activities to explore box and whisker plots. DMIMMD, Chapter 6 EMSM: page 446-449	the two interpretations the mean? How would you explain the following? Averages capture something about the data, but not everything. What can one learn about a data set from a box plot (box and whisker plots)?
Class 7	What's My Method? From *Navigating through Probability and Statistics*, Grades 3-5, NCTM, 2002. Activity: *"How many raisins are there in a ½-oz. box?"* Develop a response and provide evidence and prepare an argument to support your response. (Practice with estimation; the collection, organization, and interpretation of data; and the construction of bar graphs, line plots. Activity: Use the data collected during the *How many raisins?* activity discuss and construct stem-and-Leaf and back-to-back stem-and-leaf plot EMSM: pages 446-451	What is the role of statistical analysis in making predictions? What steps can an investigator take at each step of the statistical investigation to improve confidence in answer to the investigation question?
Class 8	Activity: Consideration of why box plots are necessary, and how using different interval lengths for box plots can give different perceptions of the data? Activity: Interpreting various data distributions and relationship to the central tendency. Compare data distributions with the same mean, the same median. Class discussion: What does the word "average" mean to you today?	What considerations need to be taken into account when comparing data sets and what can be gained by comparing data sets? What can be learned from numerical summaries of quantitative data in regards measuring the center of distribution? What can be learned from

	Compare and contrast similar data sets, including the calculation of standard deviation. DMIMMD Chapter 8 *Take Home Midterm Test*	numerical summaries of quantitative data in regards to the amount of variability within a distribution?
Class 9 and 10	Activity: Demonstrate using the class collected data (age versus years of teaching experience). Bivariate Data Scatterplots Linear regression Correlation Pairs: Use data from “foot length” and “grab a handful” (or height vs. foot length) to construct a graph. Talk about patterns, correlation versus causation, linear functions. Activity: Matching Game: Graphs, Data, Summary (VDOE-PS, page125 and page 161 EMSM: pages 444-446	How can graphs be used to examine data? What is the role of outliers in data observations? What is the strength of an association between two variables? How can bivariate distributions describe the patterns or trends in the covariability in data on two variables? What considerations need to be taken into account when making predictions using bivariate data?
Class 11	Presentations: Statistical Investigation Projects Instructors choice, use the remainder of class time satisfy any identified student needs.	
Class 12 (Sample Lesson Attached)	Introduction to Probability How Likely Is it? Activity: Between 0 and 1, VDOE-PS, page 158 Activity: Fair or Not Fair, VDOE-PS, page 165 Simple probability Theoretical vs. experimental probability	What constitutes "fair"? How is the probability of an event calculated for theoretical probability? How is the probability of an event calculated for experimental probability?

	Sample space Trials Events Relative frequency Tree diagram Fundamental counting principle Compound events Go over the guidelines for Fair Game Project. First, step for the project which will be presented during the last class. EMSM: *Elementary and Middle School Mathematics: Teaching Developmentally* Chapter 22	What is the relationship between theoretical probability and experimental probability? What does it mean to say that an event is independent?
Class 13	Activity: What's in the Bag? VDOE-PS, page 164 Activity: Mystery Bag, What's Your Prediction? from *The Super Source, Color Tiles,* ETA Cuisenaire, 2007 Activity: Determine sample space associated with tossing two or three coins; Investigate the fundamental counting principle in determining the sample space; Use tree diagram and area diagram to determine sample space; Introduce the idea of independent events. Briefly, explain the *law of large numbers.* Bring up an example – students drew 10 colored cubes from a bag and replaced them. The results were 4 red, 3 blue, 1 green, and 2 yellow. Ask – *What colors might be in the bag? How many of each?* EMSM: *Elementary and Middle School Mathematics: Teaching Developmentally* Chapter 22	What does one need to consider to determine if the probability is conditional? What is meant by mutually exclusive? What is meant by independent and dependent outcomes? How are events defined?
Class14	Activities to continue the counting principle and to introduce and explore Combinations and Permutations. The goal is to answer questions using	What is important to consider when determining if a question is related to permutations or combinations?

	counting. However, after several activities, the counting patterns may be used to investigate the rules for calculating. Activity: How Many Arrangements? From *The Super Source, Color Tiles,* ETA Cuisenaire, 2007. EMSM: *Elementary and Middle School Mathematics: Teaching Developmentally* Chapter 22	
Class 15	Presentations: Each 3-person team sets up their game from the Fair Game Project and teams rotate around the room to play the game and determine if the game is fair with support for their answer. Final Exam	

Sample Lesson Plan for Class 12: PROBABILITY

Textbook: EMSM: *Elementary and Middle School Mathematics: Teaching Developmentally,* Chapter 22: Exploring Probability
Materials: Each class activity lists materials (see attached)

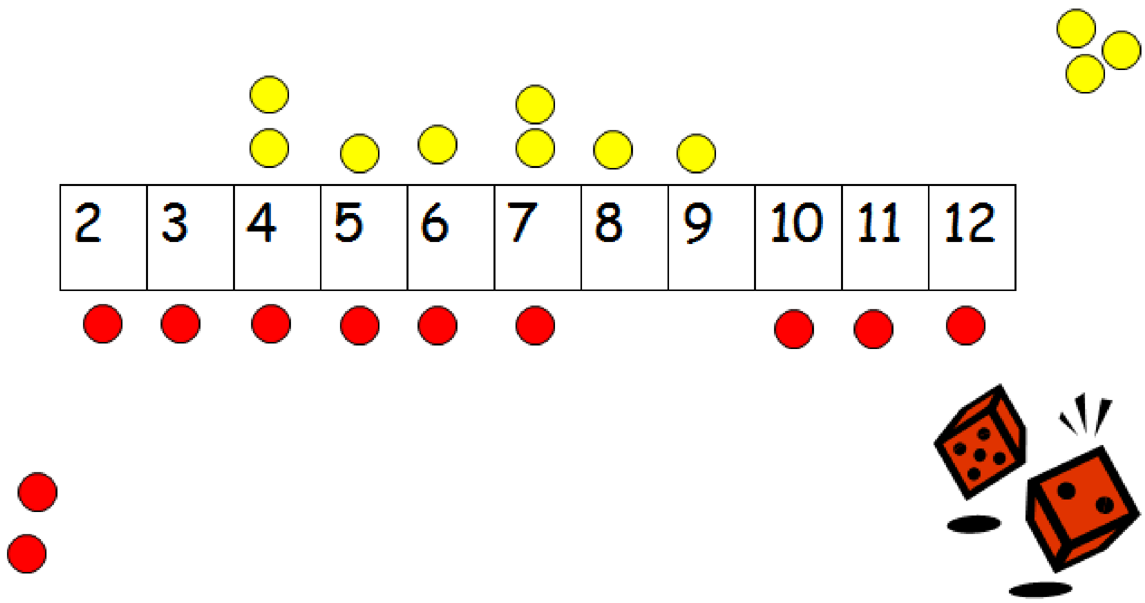
Getting Started: Today is the beginning lesson on Probability. Students will have read EMSM Chapter 22, pages 454-462. Announce the agenda for the day, goals, and objectives. (5 minutes)

Essential Questions

- Where do we see probability concepts at work in our lives today?
- What is a theoretical and experimental probability?
- How do we find both kinds of probability?
- What makes a game fair?
- What are examples of some games that are fair and not fair?
- If a game is not fair can we make it fair?

Launch (5 minutes)
How many of you have played games with someone who ALWAYS seems to win or always seems lucky? What if I told you that your friend is not lucky but is instead a math whiz? Better yet, what if I told you that I could teach you to be “lucky” as well. And if you are not the lucky type, you luck is starting to change with today because I will teach you to be that lucky person!

Two Dice Sum Game (1 ½ hours) Let students play the Two Dice Sum Game together without any discussion of probability or strategy. Each player has a number line from 2 to 12, with spaces large enough for the counters to fit on the numbers.

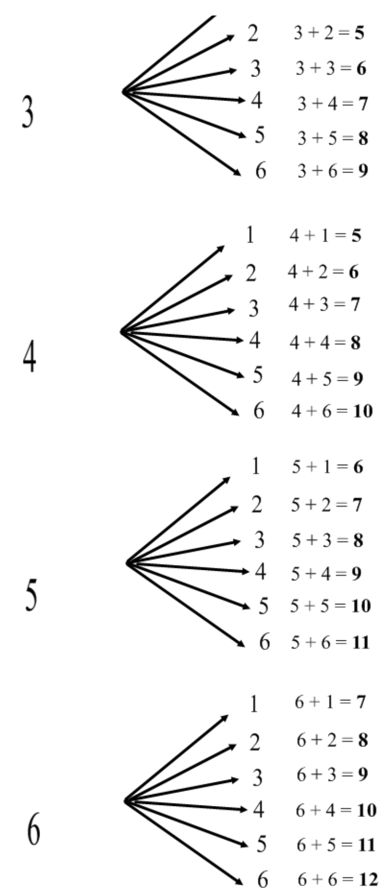


One player is red the other is yellow. Each player places his or her counters on the number line in any arrangement. (Players may put more than one counter on some numbers and none of others.) Players take turns rolling the dice. On each roll, the player who rolls removes one counter that is on the number that matches the sum on the dice. (If players have more than one counter on a number, they may remove only one.) The winner is the first player to remove all 11 counters. While students play, record the statements you hear them saying. "I can't get a 2." "It's hard to get a 12!" "Not another 7." "I keep rolling 8's!" You will be using them to summarize the findings of theoretical probability later.

After the players have completed one play, begin your discussion of probability.
What is probability? What is theoretical probability? What is experimental probability? What are the ways you find each? What is sample space? What is a tree diagram? Are there more efficient ways to find sample space other than a tree diagram? What is the theoretical probability of rolling each sum of the dice in the Two Dice Sum Game?

Table and Tree Diagram for Two Dice Sum Game

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12



Now that you have discussed some of the comments you heard while students played. Do they make sense?

Let students play the game again after calculating the theoretical probability of each sum occurring. This time, asks students to keep a tally of their rolls. When they have finished playing, (they usually need at least two plays), ask each group member to record the sum of their tallies on the whole class table you place at the front of the room.

Sample Tally Sheet

2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

Many of the students will put all of their counters on 6, 7, and 8 and then find out they still do not win. Ask them to calculate the experimental probability of their rolls. Is it close to the theoretical probability they calculated at the beginning of class? Discuss sample size and the effect it has on theoretical and experimental probability (Law of Large Numbers). Mention after the break you will participate in another activity that will help the understanding of the Law of Large Numbers.

Sum It Up! Game (30 minutes)

When students return from the break have them play the Sum It Up! Game. Students get into groups of two with a pair of dice and two cards (one blue and one yellow). On the blue card is written the following***: Scores one point if the sum of the dice is 2, 3, 4, 9, 10, 11, or 12***. On the yellow card is written the following: ***Scores one point if the sum of the dices is 5, 6, 7, or 8***. Students choose whether they want to be blue or yellow. Students roll the dice, find the sum of the dice, and record the winner. Students keep track of the score on their handout.

Students are asked to predict which color would win if the game after the dice has been rolled one time, three times, eleven times, and 21 times. Most students will pick blue each of these times just because there is so many more numbers listed on the blue versus the yellow card.

Students roll the dice **one time,** and the winner holds up their card. There will be a mixture of blue and yellow cards in the room. Students roll the dice two more times (for a **total of three**) and the winners hold up their cards. Students continue rolling until they have rolled a total of eleven times and hold up their cards. Finally, students roll a total of 21 times and display their cards. Ask, how have the cards changed? Why? What do you think would happen if we rolled the dice 100 times? Why?

The two games just played will be the subject of the article they read for lunch homework. Many of the ideas will be discussed in the reading debrief.

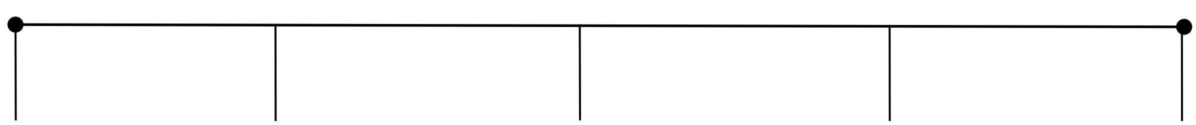
BREAK: (10 minutes)

First Experiences with Probability (30 minutes)

Debrief Two Dice Sum Game and To Sum it Up! Game. Distribute SOLs for Probability or Learning Progression for Probability. Discuss how students come to know about probability. What are their first experiences?

- **Probability Continuum** (also attached)

Probability can give you information about the likelihood of an event happening, but it is never a guarantee. The probability of an event happening can be expressed as a number from 0 to 1.



0	¼	½	¾	1
0.0	.25	.50	.75	1.0
0%	25%	50%	75%	100%
Impossible	Unlikely	As Likely as Not	Likely	
Certain				

- **Qualitative Experiences**

"How Certain Are You?" Lesson from Virginia Department of Education:
http://www.doe.virginia.gov/testing/solsearch/sol/math/4/mess_4-13ab_1.pdf

- **Quantitative Experiences** (from Van deWalle text and attached)

Using two different colors of paper plates, model the qualitative statements you made above and assign a fraction, decimal, and percent to each statement.

LUNCH (2 hours): Longer lunch is provided so that students can read their article (Aspinwall, L., & Tarr, J. E. (2001). Middle school students' understanding of the role sample size plays in experimental probability. *The Journal of Mathematical Behavior*, *20*(2), 229-245. doi:10.1016/s0732-3123(01)00066-9) and prepare individual responses to Focus Questions

Article Debrief Jigsaw (1 hour)

Student sit in table groups and count from 1-5 (number of Focus Questions). Students will gather in their 1-5 groups and discuss their individual responses to the question. They will jot down notes and serve as an expert on their question then return to share with their table groups. When the discussions at each table group have been completed, each table will be asked to share one important take away from the article or the morning experiences interacting with activities like the article with the whole group.

Two Dice Sum Game Extension Games (1 ¼ hour)

In order to allow students to apply what they have learned in new and different situations, allow them to participate in extension games (classroom copies attached).

- Two Dice Difference Game
- Two Dice Product Game
- Two Dice Prime and Composite Game
- Rock, Paper, Scissors Game
- Doubles in Monopoly Game

Before playing the games with students, have a conversation about odds and fairness. Use the example of the Two Dice Sum Game. Yes, it is fair. You put the counters wherever you choose to put them. Of course, people who know the math will be a better job at that than those who do not.

What if I changed the rules for the Two Dice Sum Game and said you could remove a counter that was on ANY ODD number if you rolled any ODD NUMBER on the dice and you can remove a counter on ANY EVEN number if you rolled any EVEN NUMBER on the dice. Is this game fair?

Students will play their extension game and prepare a poster summarizing their game. They will discuss the theoretical and experimental probabilities of the game. Provide a visual (tree diagram, chart, etc.) show how the theoretical probability was calculated. Tell if their game was fair or not. If the game was unfair, how could one make it fair?

EXTENSION GAME DEBRIEF (30 minutes)
Each group will tell about their game and share their posters.

SUIT UP! ACTIVITY (30 minutes)

Students will work individually to solve the following activity taken from:

Blair, S. & Mooney, E. (2013). Solve it: Suit up! *Mathematics Teaching in the Middle School, 18*(4), 200.

Michael randomly selected 10 playing cards from a standard deck of 52 and made the following observations about his set of 10 cards:

• From the 10 cards, 7.4 is the mean of the values.
• From the 10 cards, 8 is the median of the values.
• The mode suit is diamonds.
• The probability is .2 of randomly selecting a spade.
• The probability is .3 of randomly selecting a club or a 10.
• The probability is 1/3 of randomly selecting a 3 from among the hearts.

Assume the following values: 1 for ace, 11 for jack, 12 for queen, and 13 for king. What could Michael's 10 cards have been?

Exit Pass (5 minutes): Respond to the following question, paraphrased from the VandeWalle text, recorded on chart paper for the participants to read: Even though it takes more instructional time, actually conducting experiments and examining outcomes in teaching probability are critical in helping students address common misconceptions and builder a deeper understanding of why certain things are more likely than others. Agree or Disagree. Use examples from class where possible.

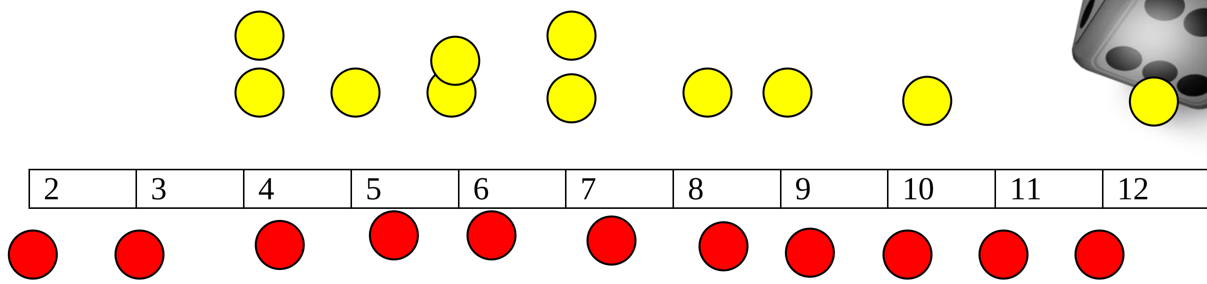
Homework for next Class (5 minutes): Come up with two more ways to solve the Suit it Up! activity. You will be assigned an article about probability from NCTM. Read your assigned article and be ready to discuss it with those who were assigned the same article tomorrow.

TWO DICE SUM GAME

You need: a partner
number line
22 two-color counters
a pair of dice

Each player has a number line from 2 to 12, with spaces large enough for the counters to fit on the numbers. One person is red the other is yellow. Each person places their counters on the number line in any arrangement. (You may put more than one counter on some numbers and none on others.) Players take turns rolling the dice. On each roll, the player who rolls removes one counter that is on the number that matches the sum on the dice. (If players have more than one counter on a number, they may remove only one.) The winner is the first player to remove all 11 counters.

Something to think about: What is the best winning arrangement of counters on the number line?



TO SUM IT UP!

Needed: A pair of dice
Two players
Blue and Yellow cards

Students get into groups of two with a pair of dice and two cards (one blue and one yellow). On the blue card is written the following***: Scores one point if the sum of the dice is 2, 3, 4, 9, 10, 11, or 12***. On the yellow card is written the following: ***Scores one point if the sum of the dices is 5, 6, 7, or 8***. Students choose whether they want to be blue or yellow. Students roll the dice, find the sum of the dice, and record the winner. Students keep track of the score on their handout.

Students are asked to predict which color would win if the game after the dice has been rolled one time, three times, eleven times and 21 times.

Most students will pick blue each of these times just because there is so many more numbers listed on the blue versus the yellow card.

Students roll the dice **one time,** and the winner holds up their card. There will be a mixture of blue and yellow cards in the room.

Students roll the dice two more times (for a **total of three**) and the winners hold up their cards.

Students continue rolling until they have rolled a total of eleven times and hold up their cards.

Finally, students roll a total of 21 times and display their cards.

Ask, how have the cards changed? Why? What do you think would happen if we rolled the dice 100 times? Why?

Standards of Learning for Probability
(Based on 2009 Virginia Standards of Learning)

SOL 2.18 The student will use data from experiments to predict outcomes when the experiment is repeated.

SOL 3.18 The student will investigate and describe the concept of probability as chance and list possible results of a given situation.

SOL 4.13 The student will:
a) predict the likelihood of an outcome of a simple event; and
b) represent probability as a number between 0 and 1, inclusive.

SOL 5.14 The student will make predictions and determine the probability of an outcome by constructing a sample space.

SOL 6.16 The student will:
a) compare and contrast dependent and independent events, and
b) determine probabilities for dependent and independent events.

SOL 7.9 The student will investigate and describe the difference between the experimental probability and theoretical probability of an event.

SOL 7.10 The student will determine the probability of compound events, using the Fundamental (Basic) Counting Principle.

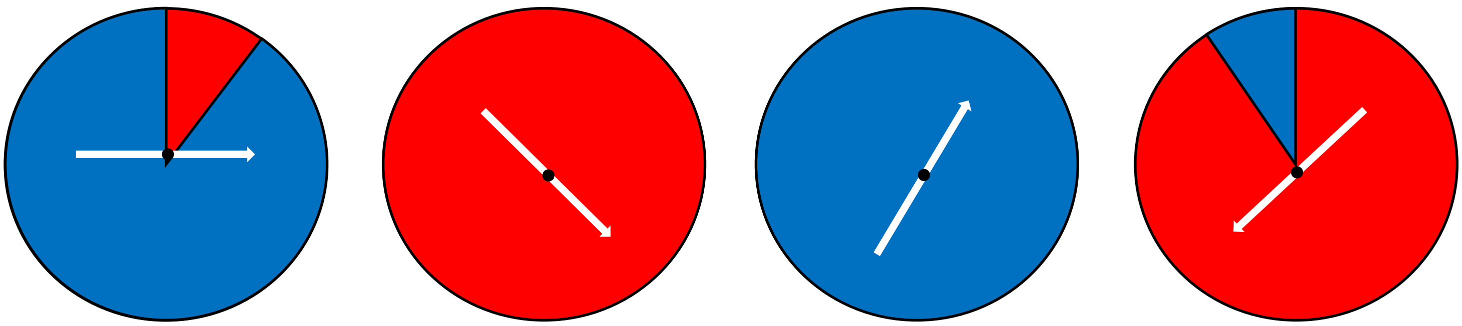
SOL 8.12 The student will determine the probability of independent and dependent events with and without replacement.

SOL AFDA.6 The student will calculate probabilities. Key concepts include
a) conditional probability;
b) dependent and independent events;
c) addition and multiplication rules;
d) counting techniques (permutations and combinations); and
e) Law of Large Numbers.

SOL AII.12 The student will compute and distinguish between permutations and combinations and use technology for applications.

PROBABILITY CONTINUUM

Look at the spinners below. Model using your paper plate spinners. For each of the models, determine the probability of the spinner landing in a BLUE space. Is it impossible, very unlikely, equally likely, very likely or certain? Place it along a continuum from 0 to 1.



Now assign a chance using a fraction, decimal, and percent. What fraction, decimal, and percent are represented by impossible? Very unlikely? Equally likely? Very likely? Certain?

Using the extra paper plates given you, make up your own quantitative probabilities and place them on the continuum.

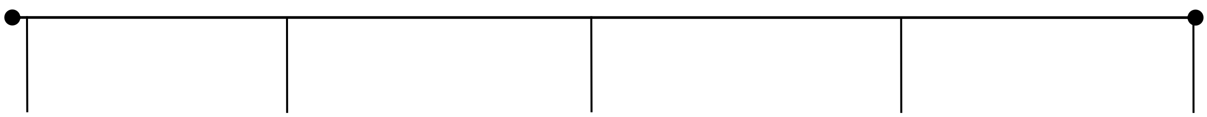
Teacher Notes:

Use the spinner faces to help students see how chance can be at different places on a continuum between impossible and certain. Connect these **qualitative** probabilities with **quantitative** amounts (fraction, decimals, and percents) and place them on the continuum according to these values.

Probability

__Probability__ is a measure used to express the likelihood of an event happening.

Probability can give you information about the likelihood of an event happening, but it is never a guarantee. The probability of an event happening can be expressed as a number from 0 to 1.



0	¼	½	¾	1
0.0	.25	.50	.75	1.0
0%	25%	50%	75%	100%
Impossible	Unlikely	As Likely as Not	Likely	Certain

FOCUS QUESTIONS FOR
Middle School Students' Understanding of the Role Sample Size Plays in Experimental Probability

1. Explain the difference between theoretical probability and experimental probability. What is the Law of Large Numbers?

2. Emily was a Level 2 prior to instruction. What in particular was her area of weakness? Were Ravi's weaknesses similar? What growth was noted in the two individuals? From your experiences with middle school aged students do you think this type of activity would generate the same type of growth?

3. How does Blake's thinking change after participating in the study?

4. Do you notice similarities in the play of the Race Game and The Two Dice Sum Game? What ideas would you want to make sure your students understand after having played this game?

5. Jot down one sentence you underlined (or that caught your attention) when reading about the game Sum it Up! Do you think your students would have similar experiences?

TWO DICE DIFFERENCE GAME

You need: a partner, a pair of dice, and a recording sheet

Players take turns rolling the dice. Player 1 gets a point if the difference is 0, 1, 2. Player 2 gets a point if the difference is 3, 4, 5. Players record the rolls in a table like the one below.

Player	Tally	Total
1		
2		

Is this game fair? Why or why not?
If the game is not fair, how can we make it fair?

If the game is not fair, how can we make it fair?

Teacher Notes / Answer Key:
Player 1 gets a point if the difference is 0, 1, 2. Player 2 gets a point if the difference is 3, 4, 5.
Is the game fair?

--	**1**	**2**	**3**	**4**	**5**	**6**
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

The game is not fair. Here are the theoretical probabilities:

$P(0) = \frac{6}{36}$

$P(1) = \frac{10}{36}$

$P(2) = \frac{8}{36}$

$P(3) = \frac{6}{36}$

$P(4) = \frac{4}{36}$

$P(5) = \frac{2}{36}$

So, player 1 will get a point 24 out of 36 times. Player 2 will get a point 12 out of 36 times. Player 1 will theoretically be awarded points twice as many times as player 2. The only way to make the game fair is to award Player 2 two points when a 3, 4, or 5 is rolled.

TWO DICE PRODUCT GAME

You need: a partner, a pair of dice, and a recording sheet

Players take turns rolling the dice. Player 1 gets a point if the product of the two numbers is odd. Player 2 gets a point if the product of the two numbers is even. Players record the rolls in a table like the one below.

Player	Tally	Total
1		
2		

Is this game fair? Why or why not?
If the game is not fair, how can we make it fair?

Teacher notes/Answer Key:
Player 1 gets a point if the product of the two numbers is odd. Player 2 gets a point if the product of the two numbers is even. Is the game fair?

X	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18

4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

The game is not fair. Here are the theoretical probabilities:

P (odd) = $\frac{9}{36}$ or $\frac{1}{4}$ and P (even) = $\frac{27}{36}$ or $\frac{3}{4}$

So, player 1 will get a point 9 out of 36 times. Player 2 will get a point 27 out of 36 times. Player 2 will theoretically be awarded points three times as many times as player 1. The only way to make it fair is to award Player 1 three points every time they roll and odd number.

Why? even x even = even
odd x odd = odd
odd x even = even
even x odd = even

TWO DICE PRIME OR COMPOSITE GAME

You need: a partner, a pair of dice, and a recording sheet

Players take turns rolling the dice. Player 1 gets a point if the sum of the two numbers is a prime number. Player 2 gets a point if the sum of the two numbers is a composite number. Players record the rolls in a table like the one below.

Player	Tally	Total
1		
2		

Is this game fair? Why or why not?
If the game is not fair, how can we make it fair?

Teacher Notes / Answer Key:
Player 1 gets a point if the sum of the two numbers is a prime number. Player 2 gets a point if the sum of the two numbers is a composite number. Is the game fair?

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

The game is not really fair. Here are the theoretical probabilities:

P (prime: 2, 3, 5, 7, 11) = $\frac{15}{36}$ or $\frac{5}{12}$

P (composite: 4, 6, 8, 9, 10, 12) = $\frac{21}{36}$ or $\frac{7}{12}$

So, player 1 will get a point 15 out of 36 times. Player 2 will get a point 21 out of 36 times. Player 1 will theoretically be awarded more points than player 2.
Since the probabilities are so close, it will take a large sample size for the game to feel unfair to participants.

ROCK, PAPER OR SCISSORS

Needed: 3 players

Do you know the rock, paper, or scissors game? All players make a fist and together say, "Rock, paper, scissors, GO!" While saying GO!, the players change their hands into one of three signals, which they then show to their opponents. The final signals are: a **Rock**, represented by a clenched fist; a **Scissors**, represented by two fingers extended; or a **Paper**, represented by an open hand, with the fingers extended and touching, in order to represent a sheet of paper (horizontal/vertical).

Normally, a rock beats a scissor, a scissor beats a paper, and a paper beats a rock but we are going to change the rules a bit and award points.

1. Play this game with two other players. Decide who is player 1, 2, and 3.

2. Play the game 25 times with the following rules:

   a. Player 1 gets a point is all players show the same sign.
   b. Player 2 gets a point if only two players show the same sign.
   c. Player 3 gets a point if all players show a different sign.

3. Tally the winning points in a table like this:

Player	**Tally**	**Total**
1		
2		
3		

4. After the game decide: is the game fair? Which player would you rather be?

5. Make a list of the ways three players could show the signs (you might consider a tree diagram).

6. Could you make the game fair (maybe change the point system)?

7. Play the game again (25 repetitions) with the new point system. Record the results. How fair is the game now?

TEACHER NOTES/ANSWER KEY:

Before starting the game, you can explain and model the activity with two pupils. The condition where Player 2 scores a point (only two showing the same sign) is often misunderstood. Ask pupils if they might have a preference for being Player 1, 2, or 3.

The list of all the different ways the players could show the signs is best found using a tree diagram like the one on the attached page.

Combined class results usually show the decided advantage goes to Player 2.

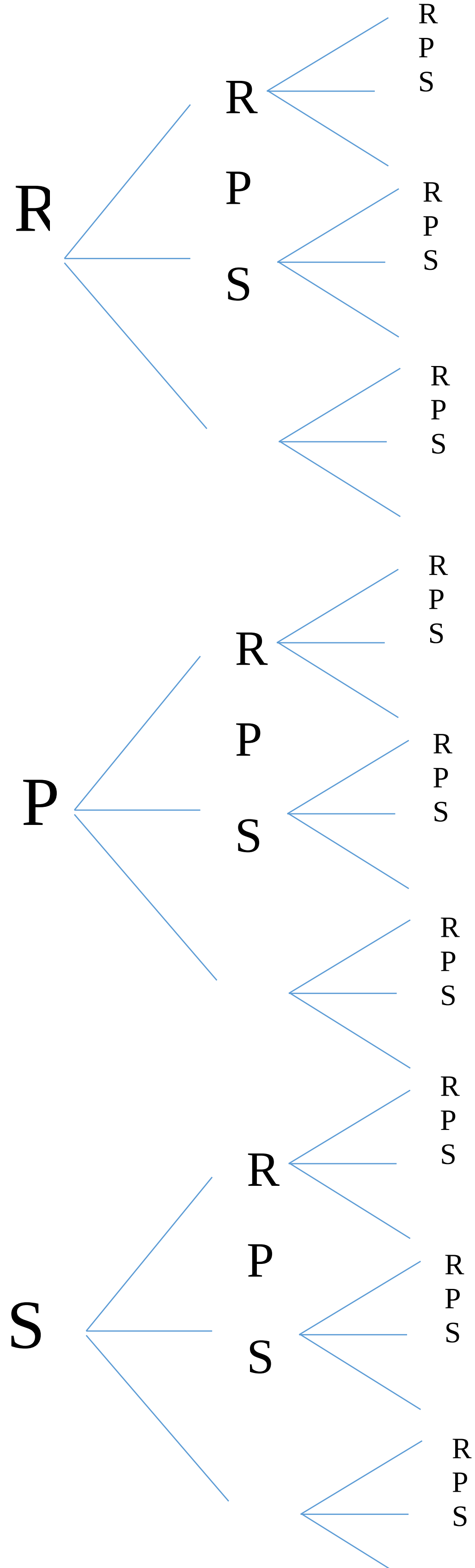
Results from playing the game 25 times might look like this:

Player	**Tally**	**Total**
1	ǁ	**2**
2	卌 卌 卌 ǀ	**16**
3	卌 ǁ	**7**

The game is not fair. It is best to be Player 2. Here is why:

Player 1	Player 2	Player 3
PPP	PSS SPS SSP	PSR
SSS	PRR RPR RRP	PRS
RRR	SPP PSP PPS	SPR
	SRR RSR RRS	SRP
	RPP PRP PPR	RPS
	RSS SRS SRR	RSP

The only way to make the game fair is to award player 1 six points each time he wins a round. Player 2 gets one point every time he wins a round. Player 3 gets three points every time he wins a round.



RRR
RRP
RRS
RPR
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PRR
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PPP
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PSR
PSP
PSS

SRR
SRP
SRS
SPR
SPP
SPS
SSR
SPS

Rolling Doubles in Monopoly

Needed: Two persons (a banker and a player)
20 $1 bills from Monopoly (or another game)
Two dice

Rolling doubles is important in Monopoly.
- You get an extra turn
  - You can get out of jail by rolling doubles.
  - You are sent to jail for 3 doubles in a row.

Do you think rolling doubles is easy?

Try this experiment:

- Give a player and the banker 10 $1 bills each.

- Have the player roll the dice.
  - If the dice shows doubles, the banker pays the player $3.
  - If the dice do not show doubles, the player pays the banker $1.

- Repeat 20 ties or until someone runs out of money.

- Record in a table like the one below.

Roll	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Doubles (Yes or No)																				
Players $1 bills																				
Banker's $1 bills																				

Do you think this "game" is fair?

Repeat the experiment with the banker paying $5 for doubles. Now is it fair?

Explain. Be sure to discuss theoretical probability in your answer.

TEACHER NOTES/ANSWER KEY:

Many students think rolling doubles is difficult because so many board games grant special privilege when you get them.

A typical game might provide a table like this:

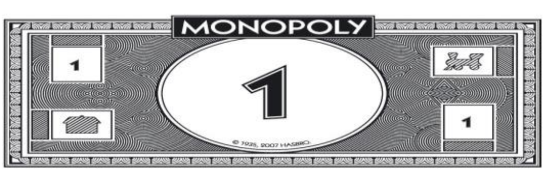
Roll	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Doubles (Yes or No)	N	Y	N	N	Y	N	N	N	N	N	N	N	N	Y	N	N	N	N	N	N
Players $1 bills	9	12	11	10	13	12	11	10	9	8	7	6	5	8	7	6	5	4	3	2
Banker's $1 bills	11	8	9	10	7	8	9	10	11	12	13	14	15	12	13	14	15	16	17	18

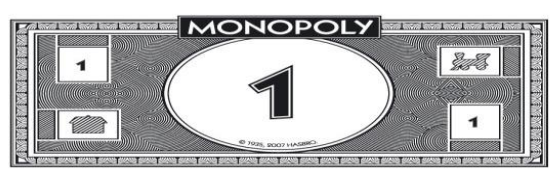
Have the player roll the dice.

- ***If the dice shows doubles, the banker pays the player $3.***
- ***If the dice do not show doubles, the player pays the banker $1.***

This experiment is not fair for the player. Combined class results should emphasize this.

Repeat the experiment with the banker paying $5 for doubles. Now is it fair?
This experiment is "theoretically" fair for both players. The chance of not rolling doubles is five times the chance of rolling doubles. But since the experimental results (in a small number of trails) seldom exactly agree with theoretical probability, combined class results should show a variety of answers.





Leadership I for Mathematics Specialists

Leadership I is a 3-credit hour graduate education leadership course designed to help prepare teachers with the knowledge and skill to become school based K-8 mathematics specialists. The prerequisite for the course is the *Numbers and Operations Course for Mathematics Specialists.* It is the first course in a series of three purposefully sequenced mathematics education leadership courses and provides a foundation for effective mathematics instruction, classroom observation, and communication. In addition, the course develops student-centered habits of mind necessary for a mathematics specialist to support teachers to become more effective classroom practitioners. The course activities and projects promote students to become more reflective practitioners as they develop their own instructional practices. *Leadership I* gives particular attention to developing the knowledge and skill of future mathematics specialists to use the state and national content and process standards to inform instructional planning, to identify research-proven best practices in mathematics education, and to design instruction that develops all students mathematical proficiency.

Course Goals:
Students in *Leadership I* advance their knowledge and skills in mathematics content pedagogy; developing, teaching, and reflecting on standards-based lessons, and diagnosing students' understanding and misunderstandings to guide instructional planning. In addition, attention is given to using a developmental progression of mathematics content, an instructional trajectory to teach a mathematics concept or skill, and the process standards in developing mathematical proficiency. Participants apply the understanding of mathematics gained during the content course(s). In summary, the course is designed around the interaction of the school student, the teacher as a learner, and the instructional program. That is:

- students as mathematics learners, including learning theory, assessment, and issues of diverse learners;
- teachers as learners, including self-reflection and collaboration as a tool for learning; and
- instruction, especially the design, teaching, and evaluation of inquiry lessons.

Course Overview
Leadership I participants examine and reflect on their own teaching in relation to current research on effective K-8 mathematics instruction. The *Standards of Learning for Mathematics for Virginia Public Schools* (SOL-M) (VDOE, 2009) with focus on the *Curriculum Framework* (VDOE, 2009) as well as national documents such as the *Principals and Standards for School Mathematics* (NCTM, 2000) and *Common Core State Standards for Mathematics* (CCSS-M) (CCSSI, 2010) are analyzed with particular emphasis given to the SOL-M Process Standards and CCSS-M Standards for Mathematical Practice.

Leadership I is highly interactive and uses cognitively demanding tasks, interactive class simulations, and school-based projects to promote participants' transfer of learning to the roles a mathematics specialist will assume in a school. Each major project and the follow-up reflection applies the student's learnings from the course, develops the course goals, and calls for the mathematics content from previous courses. The goal of the class readings, activities, and discussions are to prepare participants with knowledge and skills they will apply to carry out the class projects. The sequence of class projects informs the scope and sequence of the curriculum for the course as shown in Figure 8.

The projects assigned during the course not only develop participants content knowledge and leadership skills but also their communication skills and serve as summative assessments. Classroom presentations, written assignments, and journaling are among the formative assessment opportunities and also allow students to articulate their thinking both in written and oral communication. The information gathered by the instructors and the students during the ongoing formative assessment allows students to monitor their own learning and the instructors to monitor and guide student development. The summative and the formative assessments allow participants to assess their own leadership abilities and gain awareness of the complex nature of adult learners and the demanding role of a mathematics specialist.

In addition to the classroom discussions and the reflection papers that are required for each project instructors provide a reflection journal prompt at the end of each class to be completed and turned in at the following class. The purpose of the prompt is for students to reflect and to synthesize and apply a target concept to the role of a mathematics specialist. What follows is a brief description of each major project which is used in the student's final evaluation, supported by a rationale for requiring the assignment. A detailed set of guidelines and a rubric are provided to the students for each assignment which includes what artifacts and written papers will be submitted for a grade.

Reflection Journal

Participants make at least one journal entry each academic week the course meets to reflect on their own teaching, readings for the course, or their work with teachers and administrators. Students submit their journal entries at each class meeting. In addition, students will use these entries to develop a culminating reflection and synthesis paper.
(Rationale: Journal writing supports the course designers believe that reflection is a significant tool for personal growth and the process of reflection required to make a quality journal is a significant activity. A student's journal serves as a reference to highlight the continuum of their personal growth through the Mathematics Specialist Program coursework.)

Mathematics Autobiography

Participants begin the course by writing a 2-3 page mathematics autobiography describing past experiences in learning mathematics. This enables students to reflect on their own journey in learning mathematics and lets the instructor learn about the student, and some of their prior mathematics learning experiences. The instructions to the students are as follows.

First: Describe personal experiences, *some* of the following questions may be helpful:

- What topics in mathematics did you like, and which did you dislike?
- Who were the people who played a positive role in your mathematical life, and why?
- Who played a negative role, and why?
- Describe your good mathematical experiences and the poor experiences.

- In what environments do you learn best?
- What environments hinder your learning?
- How have your experiences as a mathematics learner shaped you as a teacher?

Second: In a separate paragraph or using bullet statements, identify your personal goals for this course.

Third: In separate paragraphs, describe your *theory* of how students learn mathematics.

(Rationale: Learning mathematics and developing as a teacher of mathematics is a journey. One's current belief's about teaching and learning is based on one's accumulated experiences; successes and challenges as well as feedback from others. In this assignment, participants reflect on their identities as mathematics learners and through writing, document their thinking which serves as a reference point for future reflections. It is important for a mathematics specialist to evaluate and honor the beliefs the teachers and principals with whom they are working hold at any given time. They also need to understand that change comes about through an accumulation of experiences that the specialist facilitate.)

Final Reflection and Synthesis Paper

Participants review and reflect upon their course portfolio of work and their personal reflection journal to evaluate and provide evidence of their individual growth; understanding the development of mathematical ideas, facilitating student learning, growing research-proven instructional practices, using an analysis of students' work to inform instruction, and assuming the role of a school-based mathematics leader. The paper should highlight ideas related to the students' work with other teachers and ideas they have considered about their own mathematics classroom instruction.

(Rationale: In lieu of a final exam this culminating project requires the student to follow the reflection prompts provided by the instructor to demonstrate their level of understanding based on the goals of the course. Participants describe how a self-selected portfolio item represents a course goal and illustrates their growth during the course. They must explain why they selected the item to illustrate a change in their thinking. The course topics that must address include: understanding the mathematics that underpins school mathematics and developing the skills and knowledge to evaluate student learning; understanding effective mathematics teaching in a way to enable interpretation of curriculum materials; planning instruction through strategic lesson design, observing a classroom lesson to evaluate student understanding; facilitating adult learners in an inquiry-based process such as a mathematics learning community; facilitating discussions based on an analysis of student work to identify learning problems and to inform instructional planning; and refining their personal philosophy of teaching and learning mathematics while building a deeper understanding of partnering with teachers and administrators to improve student achievement.)

Learning Theorist Project

Participants research an assigned learning theorist. The research identifies which learning theory is most closely associated with the particular theorist, what are some important contributions the theorist has made to mathematics education, and what lasting impact has resulted from the work of the theorist. Each student writes a 4-page paper to describe the contributions the theorist has made to teaching and learning and then discuss the more everyday school-based impact the theory has on belief about teaching mathematics. The writer will consider the various interpretations of the theorist work on their own beliefs about teaching mathematics, the question of how students learn mathematics, and how the information can assist in learning about another teacher's beliefs based on lesson design and classroom instruction. And last, consider the implications of the theory in terms of selecting curriculum materials, teaching strategies, and assessment practices. Create a PowerPoint presentation to support a 15-minute presentation to the class.

(Rationale: The goal of this assignment is to understand that everyone holds a personal belief about teaching and learning mathematics and that belief may shift over time and experiences. Mathematical beliefs can be studied in the light of major philosophical and pedagogical stances on the nature of teaching and learning of mathematics. As a mathematics specialist, it is important to recognize and respect which theory about learning has helped to shape the beliefs of the teachers and administrators. In particular, this information is helpful to understand teachers' instruction and their goals and expectations for students. Teachers beliefs and attitudes are important indicators of how open teachers may be to changing their practice or the materials they use in their practice.)

<u>Classroom Observation Project</u>

Engage a teacher at a grade level above or below the one the student is currently teaching. Meet with the teacher and review the standards-based observation protocol provided by the instructor to collect data about what students are doing during the lesson in relation to what the teacher is doing. After the observation analyze the notes using the criteria for an effective standards-based classroom. Then, based on the analysis of the data what are some next steps for this classroom to move it to a more standards-based classroom where students are making sense of the mathematics and moving towards mathematical proficiency. During a follow-up meeting use examples of student's work and comments in class to engage in discussion with the teacher.

(*Rationale: The goal of this first observation is to provide an opportunity for prospective mathematics specialists to develop their knowledge and skills to focus on students' learning and how students engage with the mathematics in the lesson by using a standards-based classroom observation protocol. The second goal for this observation activity is to analyze the data gathered during the observation and make conjectures about students' proficiencies in mathematics based on the five mathematical proficiencies (Kilpatrick & Swafford, 2002)).*

<u>Student Interview Project 1</u>

Each participant will work with a same grade-level teacher in his school to identify two students to interview that he does not teach and to jointly select a mathematics topic from number and operations for the interviews. A major component of this project is conducting the interview, analyzing the data from the interview, and identifying next instructional steps for each student. And last, the participant will meet with the interviewee's teacher to discuss the findings from the interview to discuss where the students are in their mathematical understanding about the topic and what some possible next instructional steps are for the students.

Participants will work with the instructor and other students to develop an interview protocol that will include a series of tasks and problems along with questions to explore the students' ideas and skills in regards the mathematics topic. The cases in the *Numbers and Operations* course and from the Van de Walle et al. book provide assistance in mapping out a mathematics progression for the topic and for developing the protocol.

Student Interview Project 2

Knowing and understanding the mathematical progression associated with the mathematics topic for an interview is a crucial element in developing a protocol and conducting an interview that reveals each student's thinking. Therefore, reading appropriate chapters in course texts, state and national documents, and other resources (e.g., journal articles) before the interview, and understanding the meaning of the content, is necessary

Work with a teacher at a different grade level from the first interview project and identify two students for the interview project. Along with the teacher, decide on the purpose of the interview based on the numbers and operation strand. As part of the follow-up to this project meet with the teacher to discuss findings from the interview and where the students are in the mathematical progression for the topic.
.
(Rationale: Two major threads across the three mathematics leadership courses is 1) mapping out a mathematics progression for specific mathematics topics and 2) investigating ways children think as they build an understanding of the various strands of mathematics. One way we learn about children's thinking is to listen as they articulate their own thoughts while representing and solving mathematical tasks. Interviewing student is an essential tool for mathematics specialist to learn about students understandings and to identify the root-cause of students misunderstandings. In order to gather informative data on a student, as well as a teacher, the interviewer must become adept at questioning, listening, and responding. It is also important for the specialist to understand the developmental progression for the mathematics topic and become skilled at developing the tasks and questions for an interview that will reveal students level of understanding.)

Lesson Planning Projects (2 times during the course)

The projects involve identifying a mathematics topic, a mathematics learning target for one lesson, modifying an existing lesson in a current textbook applying the ideas of the course about effective teaching, and teaching the lesson to reflect content and process goals defined in the Virginia *SOL-M* and *Curriculum Framework.* The lesson will incorporate various strategies to make mathematics accessible to a diverse set of students, taking into consideration where different students are in the mathematics progression for the topic, language challenges, etc. Use the course template adapted from the thinking through a lesson protocol (Smith, Schwan, & Hughes, 2008) to write a well-thought-out lesson plan around one or a related set of tasks or problems based on the developmental progression for the selected mathematics topic. Then teach the lesson and collect student artifacts to use in an analysis of the lesson's impact on students' achieving the mathematics learning target.

(Rationale: The goal of this project is for a prospective mathematics specialist to refine her knowledge and skills to create well-thought out standards-based lesson plans developed around task(s) or problem(s) that calls up the mathematics content and process standards defined in the state and national standards. The lesson, for one class period, should be developed around a learning target, and the mathematics content in the learning target should be well researched. The plan should demonstrate attention to how students develop understanding of the concept supporting the mathematics learning target.)

Video of Teaching Project

Plan a full class period lesson and then video yourself teaching the lesson. Use the M-Scan (Merritt, Rimm-Kaufman, Walkowiak, & McCracken, E.R., 2010). framework along with the video to analyze, without being judgmental, the mathematical learning target the lesson was planned to address, the implementation of the lesson, and the ways students interacted with the lesson. In particular, look at the quality of classroom discourse, whole class, and small group, and examine any the equity issues that may not be apparent while in the act teaching. Take risks during this project, try one or more new teaching strategies that you have seen modeled in the mathematics specialist courses as well as those encountered through readings and video in the courses you have taken so far.

The video will show the teacher working with the whole class as well as talking with small groups. Remember the goal is not to present a "perfect lesson," but to REFLECT on current practice as a mathematics teacher. The analysis of the taught lessons allows for recommendations, based on research-proven ideas, for ways to modify instruction so that all students become mathematically proficient.

(Rationale: One of the most important roles mathematics specialists assume is to support individual and small groups of teachers in developing lesson plans, reflecting on their own teaching and student learning and gaining new professional knowledge during the experience. Mathematics specialists must have a strong knowledge base of mathematics content and mathematics content pedagogy to reflect on their own teaching and that of others.)

Course Instructional Materials and Outline of Class Meetings

Listed below are the primary student and instructor texts for the course. In addition, the lists include some of the supplementary materials used by instructors to prepare for the course and some suggestions for supplementary student readings.

Student Primary Resources

Chapin, S.H., O'Connor, C., & Anderson, N.C. (2009). *Classroom discussions: Using math talk to help students learn* (Second Edition). Sausalito, CA: Math Solutions.

Kilpatrick, J., & Swafford, J. (Eds.) (2002). *Helping children learn mathematics.* Washington DC: National Academy Press. Retrieved from http://www.nap.edu/catalog/10434/helping-children-learn-mathematics.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics (PSSM).* Reston, VA: Author.

National Council of Teachers of Mathematics (NCTM). (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: National Council of Teachers of Mathematics.

Van de Walle, J., Karp, K., & Bay-Williams, J.M. (2010). (8th edition). *Elementary and middle school mathematics: Teaching developmentally.* New York, NY: Pearson Education.

Virginia Department of Education, (VDOE). (2009). *Mathematics standards of learning for Virginia public schools.* Richmond, VA: Virginia Department of Education. Retrieved from http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/2009/stds_math.pdf.

Virginia Department of Education, (VDOE). (2009). *Mathematics standards of learning curriculum framework (kindergarten - math analysis).* Richmond, VA: Virginia Department of Education. Retrieved from

http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/index.shtml
Virginia Department of Education (VDOE) Professional Development Resources for Mathematics,
http://www.doe.virginia.gov/instruction/mathematics/professional_development/index.shtml. A collection of professional materials.

Instructors Supplementary Resources

Heritage, M. (2008). Learning progressions: Supporting instruction and formative assessment. Washington, DC: Council of Chief State School Officers. Retrieved January 2014 from
http://www.ccsso.org/Documents/2008/Learning_Progressions_Supporting_2008.pdf.

National Council of Supervisors of Mathematics. (2008). *The PRIME leadership framework: Principles and indicators for mathematics education leaders*. Denver, CO: Author.
National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics.* Washington, DC: Authors.
National Council of Teachers of Mathematics (NCTM). (2014). *Principles to action toolkit.* Retrieve from http://www.nctm.org/ptatoolkit/.
Smith, M.S., Stein, M.K., Arbaugh, F., Brown, C.A. & Mossgrove, J. (2004). Characterizing the cognitive demands of mathematical tasks: A task-sorting activity. *Professional Development. Guidebook for Perspectives on the Teaching of Mathematics, Supplement*. Reston, VA: National Council of Teachers of Mathematics.

Supplementary Readings

Hess, K. (2008). Developing and using learning progressions as a schema for measuring progress. Retrieved October 2014 from http://www.nciea.org/publications/CCSSO2_KH0 8.pdf

Hodges, T., Rose, T, & Hicks, A. (2012). Interviews as RtI tools. *Teaching Children Mathematics, 19(*1), 30-36.

Knight, J. (2011). What good coaches do. *Educational leadership*, 69(2), 18-22.

Lipton, L., & Wellman, B. (2007). How to talk, so teachers listen. *Educational Leadership*, Association for Supervision and Curriculum Development.

Merritt, E., Rimm-Kaufman, S., Berry, R., Walkowiak, T. & McCracken, E. (2010). A Reflection framework for teaching math (M-SCANS). *Teaching Children Mathematics.,17*(4), 238.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics.* Washington, DC: Authors. Retrieved from http://www.corestandards.org/Math/.

Reinhart, S. (2000). Never say anything a kid can say!. *Mathematics Teaching in the Middle School*, 5(8), 478-483.

Skemp, R.R. (2006). Relational understanding and instrumental understanding. *Mathematics Teaching in the Middle School*, *12*(2), 88-95.

Smith, M.S., Bill, V., Hughes, E.K. (2008). Thinking through a lesson: Successfully implementing high-level tasks. *Mathematics Teaching in the Middle School. 14*(3), 132-138.

Smith, M.S., Hughes, E.K., Engle, R., & Stein, M.K. (2009). Orchestrating classroom discussions. *Mathematics Teaching in the Middle School, 14*(9), 548-556.

Stein, C.C. (2007). Let's talk: Promoting mathematical discourse in the classroom. *Mathematics Teacher,101*(4), 285-29.

Relevant position papers developed by the National Council of Teachers of Mathematics and the National Council of Supervisors of Mathematics: Leadership in Mathematics Education.

Websites: Videos and Other Resources
Inside Math Project, http://www.insidemathematics.org/. Provides videos of K-12 exemplary lessons being taught using the 8 mathematical practices spelled out in the Common Core State Standards as well as specific content standards at a variety of grade levels. Aligns well with Virginia Standards of Learning. The mathematical practices align with Virginia's Process Goals. The Mathematics Assessment Project, http://map.mathshell.org/materials/index.php. The Mathematics Assessment Program (MAP) aims to give life to the Common Core State Standards (CCSSM). MAP is a collaboration between the University of California, Berkeley and the Shell Center team at the University of Nottingham, with support from the Bill & Melinda Gates Foundation.

The Teaching Channel, https://www.teachingchannel.org/. Videos of K-12 exemplary lessons being taught using the mathematical practices and specific Common Core Content Standards at a variety of grade levels. Aligns well with Virginia Standards of Learning. The mathematical practices align with Virginia's Process Goals.

Course Outline: Topics and Essential Questions
The outline presented below shows the course scheduled as **8 six-hour classes.** The essential questions provided in the second column guide the selection of topics as well as the development of activities and resources identified in column 3. Column 3 shares a sampling of activities, readings, and assignments for each class as well as when major projects are assigned and due. Textbook assignments are identified using the codes below.

- **EMSTD--** *Elementary and Middle School Mathematics Teaching Developmentally Seventh Edition*
- **CDMT** -- *Classroom Discussions: Using Math Talk to Help Students Learn*
- **HCLM** -- *Helping Children Learn Mathematics*
- **PSSM** -- *Principles And Standards For School Mathematics*
- **PtA** -- *Principles to Actions*

Figure 9: Leadership I Course: Overview of Topics.

Class	Essential Questions	Activities and Suggested Resources
Pre-class assignment: Email students the guidelines for writing their mathematics autobiography and ask them to submit the assignment prior to the first class meeting.		
Class 1	What does it mean to be a community of learners?	Begin class with a team building activity such as the Broken Squares activity which can be retrieved at http://www.nsrfharmony.org/system/files/protocols/broken_squares_0.pdf. Following the team building activity establish the classroom norms for learning and working together.
Class 1	What factors influence student learning as an outcome of classroom instruction?	Use PSSM Chapter 2 and EMSTD Chapter 1 to design a group activity to jigsaw and process the six principals of school mathematics that influence student learning. This activity frames many of the upcoming activities and stimulates ideas and ongoing conversations about how best to help students gain a deep understanding of important mathematics.
	What are learning theories? Are there learning theories that are especially relevant to the teaching and learning of mathematics?	Activity and discussions that encourage students to think about learning as a process that brings together personal and socio-cultural experiences and influences for acquiring, enriching or modifying one's knowledge, skills, values, and attitudes. From this, a person develops their *personal point of view about* teaching and learning. The major concepts and theories of learning include behaviorist theories,

	What does it mean to do mathematics? What does it mean to understand mathematics?	cognitive psychology, constructivism, social constructivism, experiential learning, and situated learning theory and community of practice. EMSTD Chapter 2, pages 20-23. Supplement with mini-lecture that introduces some of the key theorists that have impacted mathematics teaching and learning. Use EMSTD Chapter 2, selected mathematics task in the chapter and the reflection questions on page 30 to have students investigate what it means to do and understand mathematics. *Project Overview and Guidelines:* • Review guidelines for the Reflective Journal. • Review the guidelines and rubric for the Learning Theorist Project *Homework:* • Reflective Journal Prompt: Read Merritt, E. G., Rimm-Kaufman, S. E., Berry III, R. Q., Walkowiak, T. A., & McCracken, E. R. (2010). A Reflection Framework for Teaching Mathematics. Teaching Children Mathematics, 17(4), 238-248. Then respond to the following prompts." - Briefly, describe the research basis used by the develops to create the M-Scans framework and what purpose did the developers have in mind for the framework. - Consider the description of each of the 8 dimensions identified in the article and now think about the various research informed frameworks and ideas that we have discussed in Leadership I. Discuss a few of the connections you can make for *each* of the dimensions. - What cautions as well as suggestions for using the document did the developers share? • Read HCLM pages 1-34 to prepare for class discussion and activity. Pay special attention to the five strands of mathematical proficiency.
Class 2	Why does a mathematics specialist or teacher leader need	Students share their learning theorist projects. Follow-up with a whole class discussion using

Class 2 Cont.	to know his/her own personal point of view about teaching and learning and also be able to discern others personal point of view? What does it mean to be mathematical literate? Why is identifying clear mathematics learning goals important? What is the role of classroom discourse in developing mathematical understanding?	EMSTD Chapter 2, page 30, For Discussion and Exploration question 2 and 3. The student will develop a way to think about and talk about being literate using the language associated with being mathematically proficient. Assign students to five groups based on the five strands of proficiency and use HCLM pages 9-16 as well as EMSTD pages 23-25 to create a poster that could be used in a grade level or PLC meeting to enlighten others. Use a gallery walk and sticky notes for peer critiques. Follow up with a whole group discussion to highlight the integrating the strands of proficiency. Select some of the questions on page 5 to connect to the work of a mathematics specialists. Use pages 12-16 in *PtA* to develop an activity for students to develop and understanding of the criteria for creating a mathematics learning goal/target and gain some experience in creating learning targets. Complete a KWL to allow students to share what they know about classroom discourse in a mathematics classroom, what they are not sure about, and what they want to understand better. Provide a set of focus questions and have students read CDMT Chapter 1 and follow-up with reading and discussing EMSTD Chapter 3, pages 43-47. Small group discussion of the focus questions. Whole class discussion: What is the role of classroom discourse in developing a community of learners? What is the role of the teacher in supporting classroom discourse? Pages 29-35 in *PtA* and the online NCTM toolkit offer options for instructors is planning. Develop knowledge and skills for developing and interviewing students as a form of formative assessment. Use Hodges, T., Rose, T, & Hicks, A.

	How can a well-defined process for interviewing students serve as formative assessment?	(2012). Interviews as RtI tools. *Teaching Children Mathematics, 19(*1), 30-36 as a discussion point. Participants will use the student interview protocol supplement to the article as a starting point for developing an appropriated student interview for the grade level students they will be interviewing. Select a video of a student interview to view in class and ask participants analyze teacher moves and flow of questions. *Project Overview and Guidelines:* • Review the guidelines and rubric for the Student Interview 1 Project *Homework:* • Reflective Journal Prompt: - Read EMSTD, Chapter 5, pages 87-88. Then think about the five strands of mathematics proficiency. How can student interview capture the students level of proficiency within a particular topic? What can a teacher learn from several student interviews that can help evaluate his/her classroom instruction? • Read CDMT Chapter 2: The tools of classroom talk, pages 11-22. Create a quick 4-column note reference chart for the 5 talk moves and the 3 productive talk formats that share what it is, why it is beneficial, what it looks like in the classroom, and how a teacher prepares students.
Class 3 Class 3 Cont.	What can be learned about the level of a student's mathematics proficiency as a result of a student interview? What are the process goals and how do they support developing mathematically proficient students? What knowledge and skills	Use a pair or triad sharing protocol to <u>debrief the student interviews project</u>. The instructor will use the opportunity to circulate and identify ideas and comments to bring out in the follow-up whole class discussion. Use *Mathematics Standards of Learning for Virginia Public Schools* pages iv.-v. and EMSTD, Chapter 1, pages 3-5 to develop a shared understanding of the five process goals/standards. Investigate the Mathematically Practices as defined in the *Common Core State Standards for Mathematics*. Jigsaw reading and small group discussion of CDMT Chapter 8, Getting Started pages 143-147. Then

	support purposeful questioning supports developing students mathematical proficiency? What should a mathematics specialist or teacher see and hear in a classroom that is discourse rich in a way that promotes mathematical proficiency?	everyone reads CDMT Chapter 8, pages 148-152. Work in small groups to review and revise three-column notes developed for homework from the previous class. Use CDMT page 171, question 2 to have students self-reflect about their own classrooms and classroom discourse. Interactive mini-lecture to process the remainder of the information in Chapter 8. Pages 35-41 in *PtA* and the online NCTM toolkit offer options for instructor's planning. Students will work in pairs. Use the Annenberg Learning online access for the Fraction Tracks video to simulate a classroom lesson. Pairs will first play the fraction track game and discuss the mathematics that students will engage in as they play the game. Then, one member of the pair will use the observation protocol to have students observe a classroom and collect evidence of the 5 talk moves and the 3 talk formats. The other member of the pair will use an observation protocol to collect evidence of process goals that are addressed in the lesson. Small group and then whole group discussion to debrief and synthesize the key ideas. Students will use these same protocols for the Classroom Observation Project. *Project Overview and Guidelines:* • Review the guidelines and rubric for the Classroom Observation Project *Homework:* • To develop an understanding of teaching with problems read pages EMSTD, Chapter 3, pages 32-43. Use the Writing to Learn questions 1-3 as focus questions to guide your reading. Underline or highlight information that resonates with you as you read the assigned pages. • Assign half the class CDMT Chapter 3, Mathematical Concepts and the other half of the class CDMT Chapter 4, Computational Procedures. Each group will use the questions at the end of their chapter to guide

		their reading. Be prepared for sharing details from the chapter.
Class 4	How can mathematics specialists use classroom observations to gain entry to teachers' classrooms and to learn about the school's mathematics program?	Use a protocol to debrief the Classroom Observation Project. Following the debrief engage small groups and then the whole group about the value and challenges classroom observations present for a mathematics specialist. Become familiar with the M-Scans framework and use a video of a classroom lesson to simulate observing a classroom lesson.
	How can instruction develop conceptual understanding and also develop computational procedures so that students are mathematically proficient?	Provide a protocol for pairs to share CDMT Chapters 3 and 4 in a way that develops the definitions, identifies the value in both goals for student learning, and what a mathematics specialist might see in a lesson plan or in a classroom observation where each is being developed in a way that contributes to mathematical proficiency. Pages 42-48 in *PtA* and the online NCTM toolkit offer options for instructor planning.
Class 4 Cont.	How is teaching through problem-solving different from teaching problem-solving?	Use the mathematics problems and the homework readings from EMSTD, Chapter 3, pages 32-43 to support students in gaining an understanding of teaching through problem-solving and the knowledge and skills to develop and implement problem-based lessons. Secure a video of a problem-based lesson to view and analyze.
	What criteria can be used to analyze and classify task based on the cognitive demand the task requires.	Review the characteristics of mathematical tasks at four levels of cognitive demand as described in Smith and Stein (1998). Pages 17-24 in *PtA* and the online NCTM toolkit offer options for instructor's planning.
	What components and structure	Use EMSTD Chapter 3, pages 47-54 to develop students' knowledge about the components and structure entailed in using the three-phase lesson

	in the lesson plan support a discourse rich task-based lesson?	format to plan a lesson. Bring a sample lesson plan for the students to analyze and discuss. Have students work in pairs to identify a textbook lesson and describe how the lesson can be converted to a problem-based lesson without drastically altering the lesson as written. Use CDMT Chapter 9, Planning Lessons to investigate the asking questions part of planning. Review the Thinking Through a Lesson protocol. *Project Overview and Guidelines:* • Review the guidelines, template, and rubric for the Lesson Planning Project 1. *Homework*: • Divide the class into 5 groups and assign each group one of the 5 content strands from PSSM. Each person will read the K-2, 3-5, and 6-8 content standards for their strand. Then write a journal reflection that compares the mathematics developmental progression defined in PSSM to the K-8 progression in the Mathematics Standards of Learning for Virginia Public Schools.
Class 5 .	What can a group of teachers engaged in a professional learning community learn from analyzing student work? What does a productive critical friends conversation about a teacher practice look and sound like?	Groups of three teachers will present three pieces of student work collected during the lesson planning project to share with the group. Instructors provide a protocol for the process and teach the class how to use the protocol. Use EMSTD Chapter 4, Planning in the Problem-based Classroom to further develop students knowledge and skills in planning for all learners. <u>Feedback Lesson Plan 1 Project</u>: Provide a checklist for pairs to use as they read and critique each other's lesson plans. Simulate a discussion between colleagues to discuss the lessons and provide feedback. Read the paper, Developing and Using Learning

Class 5 Cont.	What is the role of mathematics learning progressions in the work of teachers and mathematics specialists? How can teachers and mathematics specialists use common understandings to support thinking about and planning for increased cognitive demand and classroom rigor?	Progressions as a Schema for Measuring Progress by Karin Hess. Retrieve from http://www.nciea.org/publications/CCSSO2_KH08.pdf In small groups discuss the following questions: How would you describe a learning progression to one of the instructors? How would you describe a learning progression to a fellow teacher or you principal? What might the role of a learning progression in the work of a teacher or mathematics specialist? Develop the idea of curriculum and introduce the idea of the intended, planned for, taught, and assessed curriculum and the need for curriculum alignment. Provide information and interactive experiences to develop students' knowledge of Bloom's Taxonomy and Webb's Depth of Knowledge. The activities should provide experiences to prepare students to examine standards and identify the level or rigor called for in the standard and to examine lessons to identify the level of rigor called for in the lesson. *Project Overview and Guidelines:* • Review the guidelines, template, and rubric for the Student Interview 2 Project. • Review the guidelines for the Teaching Video Project. (This will be due the last class of the course.) *Homework:* • Journal Prompt: Starting the process of creating a quick reference guide for yourself that can help you look up quickly information about the key research-based ideas and frameworks we have discussed and used in Leadership I. This information will become a part of your leadership toolkit, whether you are a teacher working on your own teaching practice or working with a group of teachers or as a mathematics specialist or coach. To get started with the quick reference guide,

		you will develop information for the two highlighted ideas, Mathematical Proficiency and Virginia Process Goals, and the defining components.
Class 6	How can a mathematics learning progression and Webb's Depth of Knowledge framework support developing an interview protocol? What can be learned from "deconstructing" or "unpacking" a standard? How can a mathematics specialist support teachers as they move from the unpacked standard to creating learning targets for concepts and skills in standard?	Working in groups of two students will critique each other's draft student interview protocol using a set of questions provided by the instructor and the Webb Depth of Knowledge framework. Whole class activity. Use the Mathematics Curriculum Framework for Virginia Public Schools and an analytic template for unpacking the standards to work through unpacking one of the standards. In a discussion of the template bring out the role that the various components of the template play in planning for classroom instruction and assessment. Unpack the standard together. The closing discussion makes explicit the role of mathematics learning progressions, cognitive demand frameworks, and formative assessment play in interpreting and unpacking a standard. Group students in the PSSM strand activity groups as they were in at the last class. Each group will select a grade 3-5 Virginia SOL standard from the same PSSM strand to deconstruct. Each group will share with the whole group. *Project Overview and Guidelines:* Lesson Plan Project 2 and **Video of Teaching Project (Guidelines and rubric for the Video Project are included below.)** *Homework:* • Read pages 48-52 in *PtA* and the online NCTM toolkit offers options for instructors planning for activities and discussion in the next class. • Read EMSTD, Chapter 6, Teaching Mathematics Equitably to all Children. Use the questions on page 109 to guide your

		reading and prepare for discussion in the next class. • Journal Prompt: Continue the process of creating a quick reference guide for yourself that can help you look up quickly information about the key research-based ideas and frameworks we have discussed and used in Leadership I. Continue information for, the eight mathematical teaching practices identified in *Principles to Actions* and their defining components.
Class 7	How can a mathematics specialist support teachers as they move from the unpacked standard to creating learning targets for the standard's concepts and skills? What can a group of teachers engaged in a professional learning community learn from analyzing student work? What is considered as productive student struggle and how can teachers support students to gain confidence as learners of mathematics on their way to becoming mathematically proficient. How is it all connected?	Work in the PSSM strand groups from the last class and create a set of learning targets for one of the Virginia same strand standard they deconstructed in the last class. Each group will share with the whole group. Lesson Plan Debrief: Groups of three teachers will present three pieces of student work collected during the lesson planning project to share with the group. Instructors provide a standards-based protocol for the process. Develop and interactive experience for students to think about and discuss productive struggle. The activities should connect to the strands of mathematical proficiency. Pages 48-52 in *PtA* and the online NCTM toolkit offer options for instructors planning for activities and discussion in the next class. Warshuer, H.K. (2015). Strategies to support the productive struggle. *Teaching Middle School Mathematics*, 20(7), 390-393. Medoff, L. (September 2013). Getting beyond "I hate math." *Education Leadership.* Association of Curriculum and Supervision. How is it all connected? the activity allows participants to reflect on what the instructors believe are critical ideas and information and to make explicit

Class 7 Cont.		connections between them. Provided a structured activity and list of each of the primary frameworks, documents, etc. that shared "research informed foundational knowledge to support the work of mathematics specialist and teacher leaders" during the course. An important Leadership I goal is to build a foundation of research-informed knowledge to guide the work of mathematics specialist and teacher leader. Students often see these as disjointed until they begin to identify the connections. *Project Overview and Guidelines:* • Go over the guidelines and rubric for the Final Reflection and Synthesis paper. *Homework:* • Complete your quick reference guide for looking up information about the key research-based ideas and frameworks we have discussed and used in Leadership I. The entire reference guide will be submitted as an appendix to your final paper. Include the following: - Levels of Cognitive Demand Framework (Defined in Smith and Stein 1998) - Polya's Four Phases of Problem Solving - Five Practices for Using Student Responses (Orchestrating Discussions (Smith and Stein 2011)) - Framework for Types of Questions (Defined In *Principles to Actions: Ensuring Mathematical Success for All*) - Patterns of Questioning (Defined In *Principles to Actions: Ensuring Mathematical Success for All*) - Teacher Moves to Encourage Communication in Mathematics Class (Chapin, O'Connor & Anderson, 2009) - Thinking Through a Lesson Protocol (TTLP)

		(Smith, Schwan, Bill, and Hughes)
Class 8	What can a group of teachers learn when working in a professional learning community to view a video of each other's lessons? What does it mean for a mathematics leader to be guided by professionalism?	Feedback on Video Project: Simulate a critical friends group and participants will use a consultancy protocol (http://www.nsrfharmony.org/free-resources/protocols/a-z) to share 20 minutes of their video lesson from the Classroom video project (groups of 3) Use pages 99-108 in *PtA* to develop an activity to process the information in the section on professionalism. Use the ideas in NCTM's online Toolkit, link below, for Principles to Actions to reflect the ideas explored during the course and to shine a spotlight on the role of pedagogy and teacher beliefs in creating classrooms that provide an equitable learning experience for all students. http://www.nctm.org/Conferences-and-Professional-Development/Principles-to-Actions-Toolkit/Equitable-Pedagogy/

Video of Teaching Project Guidelines and Rubric for Leadership I

The purpose of this assignment is to ***reflect on your teaching*** to gain new professional knowledge from this experience. To complete this assignment, you will capture on tape a lesson that is representative of your teaching style and classroom life. You will critique your lesson and, just as a mathematics specialist is called upon to do, you will work with a small group of classmates in a consultancy capacity using a clip from each others video lesson.

Yes, it would be nice to see a “good” mathematics lesson, however, it is much more ***important to see what you learn as a result of teaching and reflecting on a lesson.*** From seeing ourselves and others, we learn how we and others appear, talk, move, interact, and teach. The video provides a window into students learning that is difficult to process during the act of teaching, maybe what we think is happening may not be what is going on in our classroom.

Viewing our video allows us to concentrate solely on what is happening during the lesson. We can see and hear the discourse. We can examine all the equity issues that may not be apparent while teaching. Moreover, most importantly, we can focus our attention on the mathematical ideas our lesson was planned to address and how the plan was enacted in the classroom.

Taping the Lesson:

The videotape will represent a small example of your classroom life and, in turn, you will be able to see the same of others who share their tapes in a small-group setting. At the Saturday class, you will present a segment of the tape and lead a discussion with your group. The small-group consultation will provide an opportunity to meet with colleagues and brainstorm strategies to build on your strengths, address weaknesses, and refine and improve your instructional practice. Conversations such as this are one of the ways a mathematics specialist provides leadership through supporting teachers to be reflective practitioners.

You are required to videotape a 30-minute continuous segment of a lesson, and submit the video to the instructors; the video will be returned to you. The 30-minute must show a lesson based on a mathematics learning goal/target and not on review for a quiz/test or administrative work. It is a good idea to complete the taping in plenty of time so that if there are issues with taping you can tape another lesson. Please preview video for picture clarity and sound before submitting. Please check that your video *is* viewable before submitting!

Your lesson should demonstrate your current practice as a classroom teacher**.** The tape should show you working with the whole class as well as talking with individual or small groups of students. **Remember the goal is not to present a "perfect lesson," but to call upon the knowledge gained in this course to reflect on your current practice as a mathematics teacher.** You will select a segment of 15 minutes from the 30-minute video that you want your small group to help you think about as described below.

In preparation for this assignment, you will need to arrange the means to tape a mathematics lesson of your choice and to have a video to bring to class. While you and your actions are of interest, please make sure that the camera operator scans the room. Capturing students is important. You will need to be able to show this 15-minute clip on a laptop computer without depending upon wireless connectivity. Your Technology Resource person should be able to help you, and perhaps it would be a good idea to talk with them as you are planning the videotaping.

Let your principal know why you are videoing and how it will be used in our course. Assure the principal that the tape will not be viewed by anyone outside your graduate course and is intended for educational purposes only. The principal will be able to let you know what if any written permissions you need to secure.

Peer Consulting Groups:

After taping, you will need to view the lesson several times and begin to develop a perspective on what you see. Assume a stance of inquiry about what happened rather than what you wished had happened. You should decide what is important and instructive for you and others to see and discuss. In a 45-minute small group consultation, you will show an **uninterrupted 15-minute segment** of the videotape, and participate in a 20-minute discussion of what was shown. The 30-minute consultation should include:

1. **Introduction** (2-3 minutes). This is a brief frame for the subsequent tape in which you explain the context of the excerpt (school, grade level, subject of the lesson, where in the unit sequence the lesson falls). Share specific questions you have that you want your peers to help you think about.
2. **Video Clip** (15 minutes). Show one continuous segment of the lesson; that is, do not skip from one segment to another. Do not talk or answer questions during the viewing. Let the tape do the talking!
3. **Discussion** (15 minutes). Ask your peers what they saw and what comments they have. In each discussion, begin using the following format:
   - **What went well in the lesson?** *Please explain: Be specific about mathematics content, children's learning of mathematics, and teaching approaches used in the lesson. Remember to make specific connections to ideas we have examined in class.*
   - **What did not go well in the lesson?** *Please explain and be specific.*
   - **What might happen differently next time?** *Please explain and be specific.*

Remember, we need to be positive but constructive, and helpful. All participants will be going through this experience and need encouragement☺

Analysis and Reflection Paper

The 3-4 page paper is a logical companion to the videotape. You will use the M-Scan framework to analyze and critique the recorded classroom lesson .

Part I: Use the attached M-Scan framework template to record your analysis and supporting evidence of your recorded lesson. Evaluate your lesson as Low Use, Moderate Use, or High Use for each dimension. Commentary should reflect your understanding of each dimension as it is described in the article as well as the research informed ideas and frameworks covered in Leadership I to date. Attach the completed template as Appendix B of the reflection paper.

Reflect on the following to prepare for writing the paper.

1. What was the mathematics learning goal(s) of the lesson? The specific mathematics content addressed in the lesson (not a SOL number of stating of the SOL)? The process goals included in the lesson?
2. What did you do to engage your students and to keep them engaged? Was this effective? Why or Why not?
3. Use precise and accurate language when talking about your knowledge of the content and pedagogy of the lesson.
4. Make some brief comments about teacher movement, giving directions, asking questions, and responding to student questions or responses.
5. Address your facilitation of discourse and any equity issues.
6. Student engagement, work, verbal responses that contributed or not to what individual students, small groups, and the whole class seemed to be learning.
7. The implementation of the lesson: (a) the extent the mathematics learning goals were achieved and your evidence, (b) any deviations you made from the written plan and

why, (c) overall effectiveness of the instructional plan, and (d) moments in the lesson you liked and disliked, and why.

8. Specific evidence of the impact of the lesson on student learning.
9. What changes you would make in the lesson and why.
10. Overall comments about what you learned from viewing the tape.
11. What would you like your peers to help you think about when you present it to them in a small-group setting?
12. What goal(s) are you setting for yourself based on the analysis of your lesson and the feedback from your peers?

M-Scan Framework (*Enhanced*)

Space will expand as you type.

M-Scan Dimension	Rating (Low Use, Moderate Use, or High Use) and Commentary
Structure of the Lesson	
Multiple Representations	
Use of Mathematical Tools	
Cognitive Depth	
Mathematical Discourse Community	
Explanation and Justification	
Problem Solving	
Connections and Applications	
Mathematical Accuracy	
[1]*Mathematical Accuracy*	• *Does the lesson consistently model appropriate and precise math vocabulary used during the lesson?* • *Does the lesson focus on bringing clarity of mathematical concepts?* • *Does the lesson demonstrate responsiveness to student mathematical thinking in such a way as to not allow for students developing misinterpretations in the current unit or later?*

[1] Mathematical accuracy was not included in the framework shared in the NCTM Journal but has been included in this enhanced framework based on authors follow-up publications and presentations about the framework.

Merritt, E. G., Rimm-Kaufman, S. E., Berry III, R. Q., Walkowiak, T. A., & McCracken, E. R. (2010). A Reflection Framework for Teaching Mathematics. **Teaching Children Mathematics**, 17(4), 238-248.

Leadership I Videotape Project Rubric

	Rating and Points Available				Points Earned
Item	**Professional**	**Proficient**	**Acceptable**	**Not Acceptable**	
Lesson Plan: Appendix A • School, grade level, length of time for mathematics • Identify the source of the lesson • Learning goal and mathematics of lesson identified.	10-9	8-7	6-4	3-1	
Videotape of lesson shows: • Mathematical task or lesson activity • Teacher/student discourse • Content knowledge and pedagogical content • 30-minute lesson continuous video tape • Quality of picture and sound was reasonable for sharing	20-16	15-11	10-6	5-1	
M-Scan Analysis: Appendix B • Evidence of understanding the criteria for each dimension • Rating provided • Evidence from the lesson video supports the rating • Precise language used in the descriptions and reflects the knowledge gained in the Leadership I course	20-16	15-11	10-6	5-1	
Peer-group consultation • Clearly communicated questions for the peer-group viewing and consultation • Identified a 15-minute continuous segment for viewing	10-9	8-7	6-4	3-1	

• Engage peer group • Provided written and oral feedback to others in group					
Reflection paper: Evidence of thoughtful reflection on the teaching activity and ability to communicate the important and critical points clearly with supporting evidence **Part 1** reflections address both the mathematics content and mathematics pedagogy including: • Your questioning, responses to students, and the orchestration of effective classroom discourse • Student questions, discourse, and their mathematical ideas that are being explored • Your knowledge of the content of the lesson • The implementation of the lesson • The impact of the lesson on student learning with evidence • Overall comments about what you learned from viewing the video • What goals have you set for your growth	30-24	23 - 18	17-11	10-1	
Organization and presentation reflective commentary • Smoothness of expression • Precision and clarity of writing • Grammatically correct, appropriate use of academic writing mechanics, follows APA format • Within page limit	10-9	8-7	6-4	3-1	

Leadership II for Mathematics Specialists

Leadership II is the second in a series of three leadership courses for mathematics specialists. It is a 3-credit hour graduate level mathematics education course designed to help prepare experienced teachers to become school based K-8 mathematics specialists. The course develops skills related to working with adult learners and deepens mathematics content and content, pedagogical knowledge. The course focuses on developing knowledge and skill to facilitate one-to-one coaching, small groups of teachers in professional learning communities, and school-wide professional development opportunities.

Leadership I, a prerequisite to this course, requires teachers to reflect on their own pedagogical and content knowledge while also studying research-based "best instructional practices" in mathematics. Leadership III which follows this course, allows the teachers to deepen and refine their knowledge and skills to facilitate the Lesson Study process, to support teachers in creating and using formative and summative assessments to diagnose student understandings and misunderstandings and to identify and use appropriate resources to address learning and teaching problems.

Course Description/Goals:

Leadership II course is designed for teachers to build those skills, understandings and dispositions required to play optimal, mathematics education leadership roles in elementary or middle schools. Prospective mathematics specialists who finish this course will:

- Develop and refine coaching skills and skills to work with adult learners.
- Develop the knowledge and skills to use data appropriately from formative and summative assessments to identify strengths and weaknesses in the school's mathematics program and to support teachers in identifying and developing plans to address student learning problems.
- Build a deeper understanding of mathematics content, content pedagogy, and mathematics learning progressions to develop standards-based lesson plans.
- Develop knowledge and skills to plan and facilitate small group and school-wide professional development.
- Become familiar with the body of research related to selected topics within the National Council of Teachers of Mathematics (NCTM) strands in mathematics education.
- Through personal reflection refine their philosophy of teaching and learning mathematics.

Course Overview

Leadership II is designed to follow Leadership I in which teachers reflect on their teaching in relation to current research on effective K-8 mathematics instruction. In Leadership I, the National Council of Teachers of Mathematics (NCTM) Principals and Standards for School Mathematics (PSSM) and Common Core State Standards (CCSS) are analyzed with particular emphasis given to the Process Standards and Process Goals. In Leadership II teachers work to develop their coaching skills and study what it means to coach adult learners and become teachers of teachers. Assignments require the teachers to practice coaching one-on-one with another teacher in their school as well as working with grade level teams. The students continue to refine their own teaching practices and expand their knowledge of current literature in mathematics education.

Course Projects

This course is highly interactive and project based. Each project is focused on the course goals and bridges the learning in class to the work specialist do in schools. What follows is a brief description of each assignment supported by a rationale for requiring the work. The instructor provides detailed guidelines and rubrics for each project.

Journal—Due Each Class Meeting

Participants will make one journal entry each academic week between. In addition, students will use these entries as one source of documentation in the culminating reflection synthesis paper. The instructor will provide additional information and a rubric.

(Rationale: Journal writing supports using reflection as the most significant form of growth and that the process of reflection required to make a quality journal is a significant activity. A student's journal provides a reference to highlight the continuum of their personal growth through the Mathematics Specialist program coursework. Entries generally reflect an experience in students' classrooms, a discussion with a peer, or some other thought that is relevant to the content and discussion from the course.)

Final Personal Growth Reflection Paper

Participants will develop a reflection synthesis paper using their journal entries, artifacts from class readings and activities, and artifacts from course projects to revisit the important ideas they have considered throughout the course. The student should highlight ideas related to work with other teachers and analysis of their own classroom instruction.

(Rationale: This culminating activity requires the student to reflect on the major themes of the course as they describe how a selected item represents a course goal and reflects their growth. They must explain why they selected that item to illustrate a change in their thinking. The course topics they must address include: Developing and Refining their Coaching skills, Building their capacity to Work with Adults in a Learning Community, Developing and Refining their abilities to Plan and Facilitate Professional Development for Larger Groups, Refining their Philosophy of Teaching and Learning, Building a deeper Understanding the Mathematics for Teaching and Becoming familiar with the body of Research in Mathematics Teaching and Learning.)

Standards-based Lesson

Each individual will use the template provided by the instructor to develop a standards-based lesson that focuses on classroom discourse and representation, teach the lesson, reflect on the lesson, and analyze samples of student work from the lesson.

(Rationale: The goal of this project is to provide an opportunity for a prospective mathematics specialist to refine his/her knowledge and skills to create well-thought out standards-based

lesson plan developed around a task or problem that calls up the mathematics content and process standards defined in the state and national standards. The mathematics learning goal should be well researched and a mathematics progression defined for the concept and skills supporting the learning goal of the lesson.)

Literature Review Project

The participants will work in groups of 3, and each person will find 3 articles related to a specific mathematics topic in the elementary or middle school classroom. Topics include: algebraic reasoning with functions and change, connections between arithmetic and algebra, developing understanding and proficiency with basic facts, base ten number system, understanding fractions as numbers, developing understanding and proficiency with adding and subtracting fractions, developing understanding and proficiency with multiplying and dividing fractions, proportional reasoning, geometry and measurement.

The components of the literature review project require:

(1) Each group member to provide a copy of their annotated bibliography and the two-page summary of each article or book chapter.
(2) Each individual to develop a 2-3 page paper that synthesizes the key ideas based on their 3 articles and addresses how these ideas inform his/her own teaching and work with other teachers.
(3) Each group will prepare a 20-minute presentation for a class presentation.

(Rationale: The goal of this assignment is to extend and deepen student understanding of a major topic in an important area of mathematics and to develop an understanding of the developmental continuum of the concept(s) and skill(s) within the topic area. This assignment also extends students' knowledge and skills for locating, reading, comparing and contrasting, identifying the main ideas, and synthesizing research. As a mathematics specialist or teacher leader, they will be called upon to find research-based information for questions posed by school staff and parents, to learn about the impact of curriculum materials or particular instructional decisions, and to help inform school-based policy decisions about the mathematics program. Students are required to use the APA format to gain experience in academic writing and documenting their work. This project provides valuable experience prior to the Mathematics Education Research course students will take the last year of their graduate program.)

Coaching Project

For this assignment, participants will plan and facilitate coaching cycle and videotape the pre-conference and post-conference with a classroom teacher. As part of this assignment, students will choose one 10-minute uninterrupted clip to present to a small group of classmates. After discussing the clip with the small group, students will develop a written summary of the important ideas related to mathematical pedagogy and content that surfaced during their own pre- and post-conferences, and a critique of their skill as a mathematics coach.

(Rationale: The purpose of this assignment is to provide an opportunity for prospective mathematics specialists to develop the knowledge and skills about content-focused coaching to support and provide professional development opportunities for mathematics teachers. As participants complete a coaching cycle with a classroom teacher, they are asked to reflect on the thinking involved in planning the preconference and post conference in terms of their own understanding of the mathematics topic of the lesson and why they developed the particular questions they posed; the important points that came up during the lesson and connections between what was said in the preconference and observed in the lesson; how well the mathematics and mathematics pedagogy were developed during the pre-conference; analysis of three students' work; what important features were brought out in the post-conference conversation and why; ideas for refining the lesson; next instructional steps; the rationale from why they choose a particular segment of the video to share; how the experience will affect them as a coach or teacher leader; what did they learn about working with another adult to plan a lesson and what do they need to learn more about to further their development as a coach. Responses to these questions require a metacognitive approach which will be a valuable practice for participants to utilize in their work.)

Planning and Facilitating Case-based Professional Development

Groups of 2-3 participants will use the template provided by the instructor to plan and facilitate a 1.5-hour professional development opportunity, in class, based on a chapter assigned from Amy Morse's *Cultivating a Math Coaching Practice.*

(Rationale: Cultivating a Math Coaching Practice serves as a tool that allows mathematics specialists and teacher leaders the opportunity to gain experience as a facilitator of professional development and to reflect upon real coaching situations such as teachers working one-on-one with a coach, coaches providing professional development, coaches working on their own practice and the decision-making that takes place within that practice. With the other class members as their audience, participants plan, implement and reflect on their assigned topic (chapter) related to the work of a mathematics coach. Presenters must think about the focus questions they will ask in small group and whole group and how these relate to their overall goals for the session. In reflecting on the lesson they are asked to share what they learned about designing and facilitating professional learning opportunities; what they learned about collaborating with a team to develop and implement professional development and what they identify as their own strengths and needs as a facilitator.)

Planning a Workshop

Students will work in pairs to develop a 3-hour workshop designed for a whole-school mathematics faculty. The workshop will be interactive, and the plan will include clearly defined goals related to the participants learning, an annotated agenda for the three hours, at least one mathematics activity, at least one reading, how the participants learning will be evaluated at the conclusion of the workshop, and a reference list indicating the research support for the activities in the workshop.

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(Rationale: This assignment has two components. The first involves collaboration with the Principal and a reflection paper about this meeting. The meeting allows the participant to

simulate a mathematics specialist discussing the school's mathematics program with the principal. Based on the conversation identify the principal's beliefs about teaching and learning mathematics.

The second component is planning a 3-hour workshop for their staff based on needs reflected in the principal's conversation. The focus must have a strong mathematics component shaped by a related literature review. A planning and annotate agenda template is provided to guide the work as most participants are just learning what it means to be a "teacher of teachers" vs. a "teacher of students." Note that this project is only the planning for a professional development session. The logistics of actually providing a workshop can be problematic as most of the participants are still classroom teachers and formal professional development time within a school is limited. It may be that after the workshop is planned and shared with the principal that time is allotted for the session, but this is entirely optional.)

Course Format and Key Activities

This course has been provided in a variety of formats. In a regular semester, the class is held for 15 three-hour sessions. Other options have been to offer 3 to 4 six-hour sessions in the summer followed by 4 to 5 six-hour sessions held about once a month on Saturdays or to hold 7 Saturday class session over the course of a school year. Each timeframe impacts the participants in different ways. During a semester class, the teacher participants are generally working full time and have the opportunity to connect their learning to their current school environment throughout the course. At the same time, this format does not allow the participants much time to reflect on each assignment or collaborate extensively with others. When the course begins in the summer and continues into the school year, there is more time for reflection and preparation for each assignment. Instructors, however, must design and assign projects appropriately. For example, implementing a lesson plan requires participants to have access to a class of students.

Regardless of the format, the course design is student-centered and inquiry based. Instructors strive to model the instructional strategies that participants are expected to implement and share with their teachers as they become mathematics specialists. Discussions often begin with individual reflection, followed by pair conversations, expanding to a small group and then whole group sharing. At times the instructors "step-out" of their teaching role and lead an explicit discussion about the teaching moves they just employed. Being able to identify and articulate these strategies is an important coaching skill for future specialists.

Though the course goals focus on building leadership skills, doing mathematics together is also an important component. Activities include solving mathematical tasks, looking at student work and considering teacher moves. Written and video cases on teaching and coaching scenarios come, primarily, from the recommended texts but other resources are introduced as necessary to meet participant needs.

The assigned projects develop participants' content knowledge and leadership skills as well as their communication skills. Many opportunities are provided for participants to articulate their ideas in writing and orally individually and in a group. This ongoing formative assessment allows instructors to monitor and guide student development and allows participants ways to evaluate their own leadership abilities and understanding of the complex nature of adult learners and the demanding role of a mathematics specialist.

Course Materials
Listed below are the primary student and instructor texts for the course. In addition, instructors will include supplementary readings such as journal articles.

Student Primary Texts
Campbell, P. F., Haver, W. E., Ellington, A. J., & Inge, V. L. (Eds.). (2013). *Handbook for elementary mathematics specialists.* Reston, VA: National Council of Teachers of Mathematics.

Morse, A. (2009). *Cultivating a coaching practice: A guide for K-8 educators*. Newton, MA. Corwin.

West, L. & Staub, F. C. (2003). *Content-focused coaching: Transforming mathematics lessons*. Portsmouth, NH: Heinemann.

Instructor Primary Resources:

Costa, A. & Gamoran, R. (2004). *Cognitive coaching: A foundation for renaissance school.* Christopher Gordon Publishers.

Hansen, Pia M. (2009). *Mathematics coaching handbook: Working with teachers to improve instruction.* Eye on Education Press. Larchmont, NY.

Kilpatrick & Swafford, Eds. (2002). *Helping children learn mathematics.* Washington, DC: National Academies Press.

Mumme, J. and Carroll, C. (2007). *Learning to lead mathematics professional development*. Thousand Oak, CA: Corwin.

National Council of Teachers of Mathematics (2000). *NCTM Principals and Standards for School Mathematics* Reston, VA: National Council of Teachers of Mathematics.

Stein, M.K., Smith, M.S., Henningsen, M.A. & Silver, E.A. (2009). *Implementing standards-based mathematics instruction: A casebook for professional development, second edition.* New York, NY: Teachers College Press.

Sowder, J. & Schappelle, B. (2002). *Lessons learned from research*. Reston, VA: National Council of Teachers of Mathematics.

Stein, M.K., & Smith, M.S. (2011). *5 practices for orchestrating productive mathematics discussions*. Reston, VA: National Council of Teachers of Mathematics.

Van de Walle, J. Karp, K. and Bay-Williams, J.M. (2010). (7th edition). *Elementary and middle* *school mathematics: Teaching developmentally*. New York, NY: Pearson Education (Allyn & Bacon). (A student text in Leadership I)

West, L, & Camron, A. (2012). *Agents of change: How content coaching transforms teaching and learning*. Portsmouth, NH: Heinemann.

Supplemental Readings

- Bay-Williams, J., McGatha, M., Korbott, B. M., & Wray, J. (2014). *Resources and tools for coaches and leaders, k-12*. Upper Saddle River, NJ: Pearson Education.
- Blount, D. & Singleton, J. (2007). The role and impact of the mathematics specialist from the principals' perspectives. *The Journal of Mathematics and Science: Collaborative Explorations*, 9, 69-77. Retrieved from http://www.math.vcu.edu/g1/journal/Journal9/JournalWork/FormattedCopies/Blount-Singleton.html.
- Carroll, C., & Mumme, J. (2007). *Learning to lead mathematics professional development*. Thousand Oaks, CA:Corwin Press.
- Chapin, S. H., O'Connor, C., O'Connor, M. C., & Anderson, N. C. (2009). *Classroom discussions: Using math talk to help students learn, Grades K-6*. Sausalito, CA: Math Solutions.
- Grant, C.M. & Davenport, L.R. (2009). Principals in partnership with math coaches. *Principal*, *88*(5), 36-41.
- Guskey, T.R. (2002). Professional development and teacher change. T*eachers and Teaching: Theory and Practice*, *8* (3), 381-391.
- Killion, J. (2010). Reprising coaching heavy and coaching light. *Learning Forward*, *6*(4), 8-9.
- Knight, J. (2009). What can we do about teacher resistance? *Phi Delta Kappan*,*90*(7), 508-513.
- Knight, J. (2011). What good coaches do. *Educational leadership*, *69*(2), 18-22.
- Lipton, L., & Wellman, B. (2007). How to talk so teachers listen. *Educational Leadership*, *65*(1), 30-34.
- Merritt, E., Rimm-Kaufman, S., Berry, R., Walkowiak, T. & McCracken, E. (2010) . A Reflection framework for teaching math (MSCANS). *Teaching Children Mathematics.*,*17*(4), 238-245
- National Staff Development Council (2006) . NSDC tool: Developing a partnership agreement between a coach and a principal 2(4), 8-9.

Course Outline: Topics and Essential Questions

The outline below shows the essential questions and a sample of activities and resources used in eight 6-hour class meetings.

Textbooks:

EMSH: *Elementary mathematics specialist handbook.*
CMCP: *Cultivating a coaching practice: A guide for K-8 educators.*
CFC: *Content-focused coaching; Transforming mathematics lessons.*

Figure 10. Leadership II course: Overview of topics.

Class	**Essential Questions**	**Topics/Resources**
Class 1	What are the skills and knowledge necessary to work with adults? What diverse roles does the mathematics specialist assume in a school? What are the potentials of grade-level meetings as professional learning communities?	Working with Adults/ North, South, East, West activity (from National School Reform Foundation) Establish class norms Multiple roles of a mathematics specialist activity. Read and discuss EMSH: Chapters 1 and 13. Instructors model facilitation of a team meeting activity structured for professional learning using Case 1 in CFC. Follow-up class discussion about planning and working with adult learners using CFC: The Foreword, the Acknowledgements, the Preface, Chapter 1 and CMCP: pages 57-78, 161-164. Project Overview and Guidelines: • Facilitation of Chapter in Morse book • Standards-based Lesson Plan (draft) Homework: • Reflective Journal Prompt • CMCP: Chapter 9
Class 2	What is content-focused coaching? What are the three phases of the coaching cycle, what is the purpose of each phase, and what is the mathematics specialist role in each phase? What does it mean to be a teacher of teachers?	Participant Team facilitates CMCP, Case 9, “Taking the Lead as a Teacher of Teachers.” Coaching, CFC Chapter 1 (3 phases of coaching cycle), Chapter 2 (model for working with one teacher- but could apply to groups) Use CFC Chapter 6, Coaching and

	As a coach, how do you have conversations with another adult about their practice? As a coach, how do you maintain a stance of inquiry and support and not judgment?	Experienced Teacher: The Case of David Younkin and the video to establish a context for studying the coaching cycle. Debrief the standards-based lesson plan and simulate a coaching pre-conference. Project Overview: • Introduce the Coaching Project Homework: • Reflective Journal Prompt based on rereading CFC: Chapter 1, 2, & 6 • Revise standards-based lesson plan and teach the lesson • Read EMSH: Chapter 3 • Read CMCP Case 1
Class 3	What is involved in preparing for and facilitating a coaching cycle? What does it mean to coach in real time? What knowledge and skills are important in reading and synthesizing research articles?	Participants facilitate CMCP, Case 1: “Observing Studying, Analyzing Planning: Preparing to Coach.” Participants facilitate CMCP Case 2: “Discerning and Responding: Coaching in Real Time.” Simulate a coaching post-conference to debrief taught lesson from the standards-based lesson planning project in the same pairs from the coaching pre-conference. Knowledge and skills to read and synthesize scholarly publications. Invite Project: • Introduce the Literature Review Project Homework: • Read EMSH: Chapter 4, pages 51-71 • Watch the post-conference for CFC: Chapter 5 New Teacher and respond to journal prompt. • Read CMCP: Chapters 1 and 2

Class 4	What does it mean to coach strategically and how is it done? What are the various coaching stances and what can be gained or lost by coaches assuming various stances in a coaching situation? How can a framework that measures student engagement be used to facilitate a conversation about a lesson and how it engages students? How can a literature review best be developed and shared?	Participants facilitate CMCP, Case 3: “Strategic Coaching: Goal-Centered Modeling in the Classroom.” Explore the various stances that a coach takes when working with teachers at various levels of expertise. Continue to develop the knowledge and skills as a mathematics specialist to facilitate grade-level teachers in creating a Standards-based Lesson. Develop the knowledge and skills as a mathematics specialist to conduct a literature review on a mathematics topic, evaluate the merit of published documents, and to create an annotated bibliography. Project: • Review guidelines for literature review project Homework: • Read CMCP: Chapters 3 and 4 • Journal Prompt about working with principal after reading CFC Chapter 8 and EMSH Chapter 14
Class 5	How can the coaching cycle be helpful to the work of a mathematics specialist? What role does negotiation and advocacy play in establishing a relationship that supports on-going communication with the principal?	Refine coaching knowledge and skills by engaging in a simulation "coaching the coach" through the sharing of a coaching video, groups of 3 using a modified consultancy protocol. Small group activity to plan for an initial conversation with a principal. Explore issues and challenges that arise when working with administrators and how to cultivate effective working relationships with administrators. (May want to use selected readings from the supplementary reading list. If possible, arrange for a panel discussion with principals and their mathematics specialist.)

	How does purpose and readiness impact planning and facilitating whole school professional development?	EMSH Chapter 5 and selected reading from supplementary readings. Plan activity to explore the decision-making process during professional development facilitation and reflect on how the purpose of the professional development impacts the decision making. Include Guskey, T.R. (2002). Professional development and teacher change. T*eachers and Teaching: Theory and Practice*, *8* (3), 381-391. Project Guidelines to Review: • Planning a Schoolwide Workshop Homework: • Journal prompt about working with principal after reading CFC: Chapter 8 • Interview the principal • Read CMCP Case 12
Class 6	What is the role of authority as it relates to school-based mathematics coaching? What facilitation moves support a positive and productive workshop?	Participants facilitate CMCP, Case 12: "Examining the Role of Authority" Presentations from literature review projects. What are socio-mathematical norms and what implications might they have for mathematics specialist facilitating professional development? Instructors model one of the presentations from the Mumme & Carrol publication, *Learning to Lead Professional Development.* Use the context to discussion planning and facilitation professional development workshops. Projects: • Planning a school-wide workshop for mathematics teachers. Homework • CMCP Case 11 • Read EMSH Chapter 2 and respond

		to journal prompt. • Assign 1/3 of the class each one of the chapters EMSH 6, 7, and 8. Prepare for a jigsaw discussion
Class 7	What strategies can mathematics coaches implement to frame a connection between their goals and teachers' goals? How can a mathematics specialist support the school's resource teachers? What criteria can mathematics specialists use to evaluate how they spend their time in terms of activities with the greatest impact on improving the school's mathematics program?	Participants facilitate CMCP, Case 11: "Framing the Connection Between Coach and Teacher Goals" Form jigsaw groups using EMSH chapters 6, 7, and 8. Each person shares important points from their chapter. Group identifies similarities and differences in the ideas shared in each chapter. Read EMSH chapter 13. Activity to identify possible "activities" a specialist might do in a school and place each on a continuum from 0 to 1. With 0 being, having no impact on teachers instructional decision making and teaching and 1 having a substantial impact. Activity to prompt reflecting across the leadership and content courses and making explicit the skills, characteristics, and resources that will help them DO the job of a specialist. Project: • Go over guidelines for final reflection and synthesis paper. • Go over guidelines for sharing the workshop planning project. Homework: • Read EMSH chapter 12
Class 8	As a "change agent" what does a mathematics specialist need to consider when working with teachers and supporting teachers with short and long term planning? What strategies might help address	Participants share their plans from the workshop planning project using a round table approach. Presentation on the challenges during the change process.

	various school mathematics program scenarios in the short term and long term? When are different types of coaching behaviors most appropriate? What are the advantages and disadvantages of each type of coaching?	Read and discuss, Knight, J. (2009). What can we do about teacher resistance? *Phi Delta Kappan*,*90*(7), 508-513 Vignettes of coaching challenges for small groups to discuss and generate ways to address in the dilemmas looking at both short term and long term impact. What does author Killion suggest constitutes coaching heavy vs. coaching light? What does this mean for mathematics specialists work with teachers?

Leadership II Sample Lesson Plan for Class 3: Preparing to Coach

Textbooks/Resources:
Morse, Amy (2009). *Cultivating a coaching practice: A guide for K-8 educators*. Corwin Press. Newton, MA.

West, L. & Staub, F. C. (2003). *Content-focused coaching: Transforming mathematics lessons*. Portsmouth, NH: Heinemann.

Framing Questions:
What is involved in preparing to coach?
What is involved in coaching a new teacher?
What does it mean to coach in real time?
What knowledge and skills are important in reading and synthesizing scholarly articles?

Welcome and Logistics (10 minutes)
Review posted agenda

Group Presentation (1.75 hours)
The assigned group will facilitate Morse's CMCP Chapter 1: "Observing, Studying, Analyzing, and Planning: Preparing to Coach." (The instructors modeled facilitating a chapter from Morse's book in Class 2. During the facilitation, the instructors "stepped in and out" to make explicit the facilitation decisions in using this prepared resource.) Teams of 2 signed up to plan and facilitate a 1.5-hour session using one of the various instructor designated chapters. Participants complete a "Grows and Glows" feedback form for the facilitators after the presentation. The instructors will use the rubric provided in the project guidelines to score the project plans and facilitation. In addition, one of the instructors meets with the facilitators during lunch to discuss the facilitation.

West Content-Focused Coaching (1.75 hours)
Chapters 1 and 2 of the Lucy West book, *Content-Focused Coaching*, were assigned for homework after the first class. Begin with a small group and then whole group discussion and review of the purposes of each of the 3 phases of the coaching cycle and the role of the mathematics specialist in each phase.

In Class 3 students will analyze *Content-Focused Coaching*, Chapter 5 on coaching the new teacher and how it differs from coaching a veteran teacher. Provide participants with the handout, included following the homework, for Chapter 5: New Teacher Kathy Sillman.

Clarify Guidelines for Coaching Project:
Review the directions for the Coaching Project highlighting the following general directions about what the activity includes:

- Planning, conducting and videotaping a pre-conference and a post-conference with a teacher around a lesson. The pre and post-conference should be no more than 20 minutes each.
- Collecting data on student engagement and learning during a classroom observation of the lesson being taught. The actual lesson that is taught by the classroom teacher does not need to be videotaped.
- Analyzing the pre and post-conference video along with any preparation notes you made, the lesson plan (this may be from the textbook), samples of 3 student's work, and any other data collected during the observation or the pre/post conferences.
- Sharing a 10-minute clip of the pre-conference or post-conference with peers in class. Choose one 10-minute uninterrupted clip of the pre-conference to share with a small group of classmates and to serve as a reference point for talking about your coaching experience.
- Writing a reflection paper about the coaching experience.

Hand out the detailed description along with the scoring rubric.

Read the seminal article by Doug Clements, "Concrete" manipulatives, concrete ideas. Contemporary Issues in Childhood Education 1 (1), 45-60. (15 minutes)

Reading the Research (30 to 45 minutes)

- Facilitate a discussion on the guidelines for reading research articles. Handout list of suggestions.
- Explain what a seminal article is and how it can be used in a literature review. Along with providing some ideas for how to gain skill for reading scholarly articles.
- Table groups discuss questions on the handout (15 min).
- Ask each table to take one question and share their insights. Ask: What did they notice about the format of the paper, the writing style, etc.
- Share the guidelines for the Literature Review Project and have participants sign up for topics from the suggested list.
- Ask participants to reread the article for homework and respond to the handout of focus questions.

Review the Literature Review Project Guidelines (10 minutes)

Homework (25 minutes)

- **Journal Entry:**
  Review the directions for the next journal entry, due Class 4, provided on a handout.
  - o Journal Prompt 1: Watch the CFC Chapter 5 (New Teacher Sillman) Post-Conference video. Write a 1-2 page double-spaced analysis and reflection on the post-conference. Speculate on what mathematics the teacher may have learned that is specific to this lesson, and to the practice of teaching and learning in general, as a result of these coaching segments. Provide support for your speculations. In addition, respond to the following, remember to use line numbers from the transcript or time marker from video to support your ideas.
    - ? How does the coach help the teacher foster student learning in the lesson coached?
    - ? How does the coach help the teacher develop teaching expertise in general?
    - ? In what ways does the coach address core questions and issues in lesson design and analysis?
    - ? How does the coach use information from the pre-conference and lesson observation to shape the post-conference?
  - Journal Prompt 2: Free write. Pick one or two ideas that have come up during the first three classes and in 1-2 pages reflect on how these ideas have helped you to expand your thinking and/or helped you to refine your teaching practice.

- **<u>Remind the group of the Projects Underway</u>**
  - o Coaching Project
  - o Literature Review Project

- **<u>Amy Morse Cultivation a Coaching Practice Presentation Project</u>**
  Remind the group that the next two Morse facilitation groups will present during Class 4. Have the groups share how they want the participants to prepare for their presentations.
  - Chapter 3: Strategic Coaching: Goal Centered Modeling in the Classroom
  - Chapter 4: Reaching a new Teacher: Math as the Conduit

- **Article Reread:**
  Ask students to reread Clement's article and use the handout "Reading Research-Based Articles: Table Group Discussions" to take notes and prepare for discussion at the next class meeting.

<u>Handout: CFC Chapter 5 Coaching the NEW Teacher: Kathy Sillman</u>
PREPARATION FOR THE COACHING EPISODE:
<u>The Context:</u>

- Read pages 48-49 in *Content-Focused Coaching* and think about:
  - o What do you know about the teacher? (consider background,

knowledge of mathematics, inquiry level)
- What do you know about the class of students? (grade level, beliefs, and attitudes about mathematics and learning mathematics)

- Do the "math" in the lesson yourself before you read any further.

> ***You have eleven fruits in your basket. Some are one kind of fruit and the rest are another kind. How many of each could you have?***

- What are the big ideas or key mathematics concepts that this problem can support?

- As K/1 students work on this problem what representations do you think they may use? what 'noticings" might emerge? What combinations are they likely to find?
- Review the Goals and Features of Content-Focused Coaching and Guide to Core Issues in Mathematics Lesson Design. Think about where the emphasis might be with a new teacher.

NEW TEACHER: VIDEO OF LESSON
For this case, please watch the lesson first, you also have the transcript of the lesson .
Reminders as you view the video of the lesson:
- ❖ What do you want to focus on or pay attention to as you watch this video?
- ❖ The purpose of viewing is to analyze what actually happened, not to focus on what the teacher should have done.
- ❖ Collect specific evidence of how students are making sense of the mathematics or not making sense of the mathematics. Pages 61-67 in CFC book offer some additional insights.

What did you notice—complete this column as your watch the video and review the transcript	How would you bring this up in the Post-Conference

1. After viewing the video go through the transcript and make any additional notes about the lesson.
2. Go back through what you wrote down and rank order what you would bring up first, second, etc. in the post-conference.
3. In the second column generate some ways to bring this up in the discussion and what evidence you will share with the teacher when this comes up.
4. What might you do differently if this were a veteran teacher?

New Teacher: Preconference Video
While it is not possible in a school to watch a lesson and then have a preconference, we are going to that now. This is a unique opportunity to compare the taught lesson to the ideas and agreements brought out in the preconference.
Reminders as you view the Preconference video. You also have a transcript of the preconference.

- Focus on "what was done" NOT of the things you would have done
- Notice where you feel uncomfortable and ask
  - What the purpose of the coach's move might be?
  - Whether you agree with the perceived purpose?
  - How might you accomplish that same end in your own style?

TIPS for 'focusing' during your preconference viewing:

- Think about the "Guide to Core Issues in Lesson Planning."
- Select a couple of questions to focus on when you listen to the dialogue
- Listen for references in the dialogue to that section
-

Watch the preconference and use the graphic organizer below to take notes. Then answer the questions following the chart.

Invitations to Teacher to Verbalize Their Perceptions (thoughts, plans, deliberations, arguments)	***Coach Provides Direct Assistance***

Additional information about the pre-conference can be found in the *Content-Focused Coaching* book pages 52-61.
Make some notes about the questions below to prepare for the group and whole class discussions.

- When is the coach offering suggestions?
- When is she soliciting ideas from the teacher?
- Who seems to be guiding this session? What coaching behaviors based on the Continuum of Coaching Moves is evident in this session/
- Who is making the final choices about what will and will not be included in the lesson?
- As you think about your analysis of the lesson what connections do you see to the preconference? Are there changes you would make to the order in which you would bring things up in the post conference, are there things you would add?
- What similarities and differences do you note between coaching a new teacher and coaching a veteran teacher?

Handout: Reading Research-Based Articles
Clements, D.H. (1999). 'Concrete' manipulatives, concrete ideas. *Contemporary Issues in Childhood Education, 1*(1), 45-60.
As a table group discuss the following and make notes. Each table will be called upon to facilitate a discussion of one the questions with the whole group.

What seems to be the purpose of the abstract?	
What kind of journal article is this in terms of research?	

What did you learn from the introduction?	
What did you learn from the headings and sub-headings?	
What is the main hypothesis? Why is this research important?	
What are the main themes of this article?	
Are there any conceptual/theoretical frameworks presented?	
What questions were raised for you by reading this article?	
What might be the next step after reading this article?	
What did you learn from the reference section?	

Leadership III for Mathematics Specialists

Leadership III is the last in the series of three mathematics education leadership courses for mathematics specialists. It is a 3-credit hour graduate education course to prepare teachers with at least 3-years of classroom teaching experience to become school based K-8 mathematics specialists. In the course students continue to develop the knowledge, skills, and tools to partner with the principal to lead the school's mathematics program, and to coach teachers to improve their classroom practices. The course focuses on leadership for the school-based mathematic program components; instruction, curriculum, and assessment. Students advance their abilities as they learn to facilitate the lesson study process as a PLC activity, create and use formative and summative assessments to diagnose student understandings and misunderstandings, and using data to identify and to address student learning problems and classroom instructional problems.

Course Description and Goals:
Leadership III is designed to build those skills, understandings, and dispositions required for optimal mathematics education leadership roles in elementary and middle schools. Prospective mathematics specialists who finish this course will:

- Apply the knowledge and skills developed in Leadership I and II to facilitate the lesson study process.
- Apply the knowledge and skills developed in Leadership I and II to develop a standards-based unit of instruction.
- Develop and apply the knowledge and skills to create formative and summative assessments and to analyze the resulting data.
- Apply the knowledge and skills developed in Leadership I to use data and student work artifacts from formative assessments to diagnose student understandings and misunderstandings and determine next instructional moves.
- Develop and apply the knowledge and skills needed to carry out formative classroom instructional observations and to coach teachers to refine their mathematics instruction to improve student learning.
- Develop and apply the knowledge and skills to facilitate data discussions to identify learning problems and their underlying causes, to establish SMART goals and action steps to address those problems, and to evaluate the effectiveness of the action steps in terms of student learning.
- Develop and apply the knowledge and skills to evaluate curriculum support materials for the school's mathematics program.
- Develop and apply the knowledge and skills to enable communication and formal presentations to stakeholder audiences; parents, teachers, school division administrators, etc.
- Develop advocacy and collaboration skills to support partnering with the principal in leading the school’s mathematics program.

Leadership III Course Overview
Leadership III continues to cultivate the knowledge and skills necessary to assume the role of a mathematics specialist and to apply and extend what students learned in Leadership I and II. In Leadership I participants examine and reflect on their teaching in relation to current research on

effective K-8 mathematics instruction. The *Virginia Standards of Learning for Mathematics* (SOL-M) (VDOE, 2009) with focus on the *Curriculum Framework* (VDOE, 2009) and national documents such as the *Principals and Standards for School Mathematics* (NCTM, 2000) and *Common Core State Standards for Mathematics* (CCSS-M) (CCSSI, 2010) are analyzed with particular emphasis given to the SOL-M Process Standards and CCSS-M Standards for Mathematical Practice. In Leadership II students examine the various roles that mathematics specialists assume in schools and in particular develop their coaching skills; learn what it means to coach adult learners and become teachers of teachers. In addition, participants continue to refine their own teaching practices and expand their knowledge of current research literature and effective practices in mathematics education.

Course Projects

This course is highly interactive and project based. Each project is focused on the course goals and bridges the learning in class to the work specialist do in schools. In addition, each major assignment and the follow-up reflection develops the course goals and calls upon the content, pedagogical content, and leadership knowledge from previous courses, including content courses. What follows is a brief description of each major assignment supported by a rationale for requiring the work. The instructor provides detailed guidelines and rubrics for each project.

Reflection Journal

Participants make one journal entry each academic week the course meets to reflect on their own teaching, readings for the course, or their work with teachers and administrators. Instructors may provide prompts for some entrees. Students submit their journal entries at each class meeting. In addition, students will use these entries to develop a culminating personal growth reflection paper. The instructor will provide additional information and a rubric.
(Rationale: Journal writing supports the course designers' belief that reflection is the most significant form of growth and that the process of reflection required to make a quality journal is a significant activity. A student's journal provides a reference to highlight the continuum of their personal growth through the Mathematics Specialist Program coursework. Entries generally reflect an experience in their classroom, a discussion with a peer, a reading or some other thought that is relevant to the content and discussion from the course.)

Final Personal Growth Reflection Paper

Participants review and reflect upon their course portfolio of work, in class and outside of class activities, readings, and projects, and their personal reflection journal to evaluate and provide evidence of their individual growth and understanding about mathematical ideas, student learning, coaching, teaching practices, assessment and data practices, and role of a school-based mathematics leader. In their discussion, a student should highlight ideas related to their work with other teachers and ideas that they have considered about their own mathematics classroom instruction. The instructor will provide additional information that includes the dimensions to be address in the paper as well as an evaluation rubric.
(Rationale: The culminating project serves as the final exam for the course. Students must address the dimensions in the prompts, based on the goals of the course, provided by the instructor to demonstrate their level of understanding of the course learnings and of connections

among those learnings. Participants describe how self-selected portfolio items illustrates their growth.)

Professional Book Review

Select a professional book to read and critique from the list of current mathematics education books provided by the instructor. Then develop a 2-3 page review of the book related to how the resource may be used and shared with teachers and other mathematics specialists. In addition, prepare and deliver a 5-minute oral presentation to communicate how this resource may be used in the work of a mathematics specialist. Additional guidelines and rubric will be provided by the instructor.

(Rationale: The goal of this assignment is to become a critical consumer of professional materials and an effective communicator of information which are important professional responsibilities for a leader in mathematics education. Mathematics specialists need the knowledge and skills to evaluate information presented about commercial resources such as professional books and evaluate the value of the resource in light of an identified need. They must be able to effectively communicate information to various audiences; teachers, parents, and community members, for a variety of purposes, including school improvement planning, study groups, lesson study, etc.)

Lesson Study Project

The Japanese lesson study model is a comprehensive research-based professional development practice that applies much of the knowledge and many of the skills that support teachers working together in a professional learning community to improve their instructional practice. Each participant will work with 2-3 classmates to complete a four-stage lesson study cycle (goal setting, research and planning, lesson implementation and data gathering, reflection and lesson revision). Detailed guidelines and a rubric will be provided by the instructor.

(Rationale: The goal of this assignment is to provide participants with the opportunity to experience Japanese lesson study as way to engage teachers in a professional learning community. A school-based mathematics specialist is a change agent embedded in the school to facilitate teachers and administrators growth in identifying student learning problems and in transforming instruction to increase student learning. To do this successfully, a specialist must acquire the knowledge and skills to lead practice-based professional development that fosters deeper mathematical understanding, pedagogical change, collaborative work, and a reflective practice. This assignment enables the participants to work collaboratively with team and to assimilate and apply what they have learned in their mathematics content and in all three leadership courses. In addition, this assignment embeds a simulation of the three stages of coaching; pre-conference, observing the lesson being taught, and post-conference. The course instructor attends the lesson implementation and reflection which provides an opportunity to evaluate the growth of the participants.)

Classroom Diagnostic Observation Project

The goal of this project is to develop the knowledge and skills for a mathematics specialist to plan and carry out a classroom observation cycle following the coaching model; preconference, lesson observation and data collection, and post conference. During a classroom observation cycle, the mathematics specialist and the teacher meet for a preconference, during which the terms of the classroom observation are established. Based on the goals set in the preconference select from and use one of the observation instruments provided by the instructor. The instrument will be used to collect data to engage the teacher in a post-observation conversation. Each participant will use the instrument in at least 2 different grade level/course classrooms. In addition to the following steps detailed guidelines and a rubric will be provided by the instructor.

- Identify two classrooms at different grade levels that you wish to observe.
- Preconference: Meet with the teacher to identify goals for the observation.
- Choose one of the three data collection forms based on the preconference to use during the classroom observation: (1) Learning and Pedagogy, (2) Math Content or (3) Intellectual Community Observation Guide from *Lenses on Learning*[1].
- Review the observation guide carefully and plan how you use the guide to collect data paying attention to the teachers identified goals.
- Take notes during the observation noting specific things that students or the teacher are saying, writing, and doing. This data will be used in the post-conference.
- Review the notes from the observation and develop one or two things that you want to pursue in the post-conference reflective of the teachers' goals. Think about how you will bring them up and what adjustments in instruction may strengthen student learning.
- Take notes during the post conference. Very soon after the conference review and flesh out your notes.

Use the project guidelines and rubric to prepare the required materials to submit.

(Rationale: The work of a mathematics specialist is at the intersection of student learning and the instructional design and delivery to support students' learning mathematics content. To achieve the goal of mathematics specialists to increase teacher effectiveness a specialist must be skilled in gathering data about the impact of instruction on student learning. Specialists sometimes depend on informal observations to survey the school's instructional program and gain an overall view of what students and teachers are doing in classrooms. However, a diagnostic observation is a more purposeful and intentional formative observation of a classroom necessary to diagnose particular learning challenges. The data collected during the observation provides specific evidence of student engagement and developing understanding during a lesson. Specialists with knowledge of the kind of data to collect during a classroom observation and of the protocols for collecting that data are able to identify contributing instructional factors to student learning problems and to build a productive coaching relationship between the teacher and the specialist. In order to increase a teacher's content and content pedagogical knowledge the specialist needs the knowledge and skills to gain access to a teachers' classroom, collaborate with a teacher to identify challenges in her classroom, develop an observation guide to collect data that will illuminate the challenge during a lesson, and then to facilitate a conversation to examine the data and develop ways the specialist can support the

[1] The forms are included in the *Lenses on Learning Supervision: Focusing on Mathematical Thinking, Facilitator's Guide* by C. Grant, B. S. Nelson, E. Davidson, A. Sassi, A. S. Weinberg, and J. Bleiman.

teacher to adopt more effective instructional strategies. This project provides valuable experience for participants to stimulate and carry out a diagnostic observation cycle in a school setting. It also provides an opportunity to observe students at grade levels different from the participant's grade level.)

Working with Proficiency Gap Sub-groups

The “achievement gap” in education refers to the disparity in academic performance between groups of students. The achievement gap is a learning problem in many schools and shows up in student grades, standardized test scores, grade level grouping and course selection, dropout rates, and college completion rates, among other success measures. The achievement gap, as it is usually reported is the difference between the percentages of students that are proficient in each sub-group of students

The Virginia Department of Education considers the reading and mathematics performance of students in three “proficiency gap groups” comprising students who historically have had difficulty meeting the state’s achievement standards.

- Proficiency Gap Group 1 – Students with disabilities, English language learners and economically disadvantaged students, regardless of race and ethnicity
- Proficiency Gap Group 2 – African-American students, not of Hispanic origin, including those also counted in Proficiency Gap Group 1
- Proficiency Gap Group 3 – Hispanic students, of one or more races, including those also counted in Proficiency Gap Group 1

The purpose of this project is to learn more about classroom students assigned to particular proficiency gap groups and how to best support the mathematics learning for each sub-group of students. Each team or 3 is assigned one of the proficiency gap sub-groups that follow. Each team researches their sub-group to learn about the 1) potential challenges to learning mathematics and 2) research-based best practices for supporting students in the group in learning mathematics. The group will develop a *Quick Reference Guide* to help mathematics specialists and teachers become aware of potential challenges the sub-group faces in a mathematics classroom and find suggestions for ways to help these students improve their mathematics learning.

- Proficiency Gap Subgroup 1 - Students with disabilities
- Proficiency Gap Subgroup 1 - English language learners
- Proficiency Gap Subgroup 1 - Economically disadvantaged students
- Proficiency Gap Subgroup 2 - African-American students
- Proficiency Gap Subgroup 3 - Hispanic students

In addition to sharing the print Quick Reference Guide with everyone in class, each team will provide a 15-minute presentation about the information in the guide. The overview should inform the class about the potential instructional and learning challenges and provide suggestions for ways to help these students improve their mathematics learning.

(Rationale: Often because of state and national assessments mathematics specialists are called upon by administrators and teachers for assistance in improving the achievement levels of gap-group students faced with challenges to learning mathematics. The groups gaining most attention in schools typically fall into categories that can be identified as cognitive,

socioeconomic, second language learners, or racial/ethnic. More importantly, the mathematics specialist is the leader for equity for all learners in the school's mathematics program which goes beyond increasing test scores. Ensuring equity means encouraging and supporting teachers and the administrators to set high expectations, ensure effective learning opportunities, make appropriate accommodations that maintain high levels of learning, and support every student to reach high levels of learning. The Proficiency Gap Sub-groups project provides an opportunity for participants to develop the knowledge to be able to evaluate instructional resources and programs, differentiate learning activities, and create support structures for each identified sub-group. An extension of the project presentations asks participants to compare recommendations for the various sub-groups and to identify any generalizations about instructional design and delivery to support student learning. This discussion brings out the value of high levels of student cognitive engagement; new learning built on students' prior knowledge; scaffold learning to make connections to concepts, procedures and understanding; classroom discourse where are expected to explain their thinking; and problem-solving that is supported with multiple representations and multiple strategies.)

Student Diagnostic Interview Project

This is a two-part assignment, to plan and conduct a diagnostic interview with struggling student to evaluate their understanding of a particular mathematics concept or skill in the patterns, functions, and algebra strand. Then use the interview data as a formative evaluation to develop an instructional plan to address the student's misunderstandings and to move them forward in the mathematics developmental continuum for the topic. In addition to the following steps detailed guidelines and a rubric will be provided by the instructor.

(Rational: Mathematics specialists coach teachers to help them develop their practices in instruction and in formative assessment. Effective teachers use informal and formal formative assessment to determine students' understandings and misunderstanding to adjust their instruction and plan interventions to address misunderstandings. The diagnostic interview project enables participants to experience working with a classroom teacher to figure out how to help a struggling student. In addition, by reviewing the interview protocol with the teacher along with the student's responses and then discussing next instructional steps develops the teacher's practice. The specialists coaches the classroom teacher in what to look for and listen to in the classroom to continually monitor students understanding and how to adjust instruction. The project develops the participant's knowledge and skills to understand mathematics in a connected way that informs developing an interview protocol, to listen to and interpret what children are saying about their thinking, and to develop an instructional trajectory that moves students from where they are to the next step in the mathematics developmental continuum.)

Decision-Making with School Data Project

The purpose of this assignment is to develop knowledge and skills to collect and analyze data to support school-based data conversations for the purpose of identifying and substantiating student learning problems in mathematics and for developing an action plan to address an identified problem. The structured processes and templates in the project are discussed and illustrated in the course text, *Using Data to Improve Learning for All: A Collaborative Inquiry Approach* (Love, 2009). Detailed guidelines and a rubric will be provided by the instructor to elaborate on the following steps.

Step 1: Meet with the principal to review the project and to discuss what she believes are some of the school's probable learning problems in mathematics. As part of this discussion with the principal identify the various kinds of assessment data available and how to access the data.
Step 2: Based on the problem you want to investigate further gather the following data and display the data in the Excel spreadsheet template provided by the instructor. There is one Excel sheet for items a and b; feel free to add as many additional pages as you would like for item c. This information will be submitted as an attachment to the final paper.

- School demographic data
- Disaggregated Virginia Standards of Learning Assessment data for the grade level or subject that will help to investigate the learning problem
- Individual Choice: Identify at least two additional data sources and create sheets in the spreadsheet for this data. The conversation with the administrator about data available in the building can guide this selection.

Step 3: Use the graphing feature in Excel to develop charts and graphs for the data and for various combinations of data to compare and examine the relationship between different sets of information. Use appropriate displays for any data that cannot be graphed. One copy of each graph or display will be attached to the final paper. A minimum of six graphical displays is expected.
Step 4: Review the data displays and make notes about what you are noticing and what trends, patterns, gaps, etc. seem to be emerging. During this step you are not looking for reasons, just looking for the "story" revealed by the data. Also, think about what additional data/information is needed to further develop your ideas about a student learning problem. These notes will help you prepare for the collaborative data discussion.

Questions to guide analyzing the data:

- What important points seem to pop-out?
- What are some patterns or trends that are emerging?
- What seems to be surprising or unexpected?

What makes a good statement?

- Statements should communicate a single idea about student performance.
- Statements should be short and clear.
- Statements should incorporate numbers.
- Statements should focus on just those direct and observable facts that are contained in the data, without interpretation or inference?
- Statements may use relevant data concepts; mean, median, mode, range, or distribution?

Step 5: Make a copy of the table, Identifying Student Learning Problem: Summary of Findings, located on page 62 in *Using Data to Improve Learning for All: A Collaborative Inquiry Approach* (Love, 2009) to use following the data discussion to identify a student learning problem. Record your analysis by the level of data available from the different data sources used in step 4. In this table, you will record your “noticings” but do not make any conjectures as to

why. This step will reveal any needs for any additional types of data or the need to drill down further into the data. The table will be submitted as an attachment with the final paper.
Step 6: Identify four of the data displays or graphs created in step 2 to reproduce on chart-size paper to use in a class simulation of a data discussion with your peers. During this simulation, you will be the facilitator.
Step 5: Refer to the project guidelines and rubric for instructions on developing the final paper for this project.
(Rational: A mathematics specialist must be able to work with administrators and teachers to use various data related to student learning, including the results of district and state assessments, summative classroom assessments, benchmark common assessments, formative common assessments, and formative classroom assessments for learning to identify and address student learning problems. The mathematics specialist needs the knowledge and skills necessary to understand the kind of data that can be collected from different types of assessment, to secure data at various levels appropriate to the decision-making goal, to understand and interpret data and be able to organize and display the data so that analyzing it is a manageable task. This initial work with the data helps the specialist plan for meetings with the principal and the teachers. Acting as a data coach, the specialist facilitates the collaborative inquiry into the data; using the results from the various assessments to drill down to identify student learning problems. Next, the specialist facilitates the groups' discussion and further research that leads to identifying the underlying reasons for the student learning problems, setting SMART goals to address the learning problem, and developing an action plan to address the learning problem. This is information is used by the specialist to set goals for coaching and other professional development activities with teachers.)

Course Outline and Instructional Materials

Listed below are the primary student and instructor texts for the course. In addition, some of the supplementary materials used by instructors to prepare for the course and some suggestions for supplementary student readings are listed.

Student Primary Resources

Campbell, P. F., Ellington, A. J., Haver, W. E., & Inge, V. L. (Eds.). (2013). *Handbook for elementary mathematics specialists.* Reston, VA: National Council of Teachers of Mathematics.

Love, N. (Ed.) (2009). Using data to improve learning for all: A collaborative inquiry approach. Thousand Oaks, CA: Corwin Press.

Stein, M.K., Smith, M.S., Henningsen, M.A., & Silver, E.A. (2009). Implementing standards-based mathematics instruction: A case for professional development, Second edition. Reston, VA: National Council of Teachers of Mathematics. (This is a student text in Leadership I).

Van de Walle, J.A., Karp, D.S., & Bay-Williams, J.M. (2010). *Elementary and middle school mathematics teaching developmentally seventh edition.* Boston, MA. Allyn & Bacon. (This is a student text in Leadership I)

Wiliam, Dylan. (2011). Embedded formative assessment. Bloomington, IN: Solution Tree Press.

Instructors Supplementary Resources

Grant, C. M., Nelson, B. S., Davidson, E., Sassi, A., Weinberg, A. S. & Bleiman, J. (20060. *Lenses on learning supervision: Focusing on mathematical thinking, Facilitator book.* Parsippany, NJ: Dale Seymour Publications, Pearson Group.

Heritage, M. (2007). Formative assessment: What do teachers need to know and do? Phi Delta Kappan, 89(2), 140–146.

Heritage, M. (2008). Learning progressions: Supporting instruction and formative assessment. Washington, DC: Council of Chief State School Officers. Retrieved January 2014 from http://www.ccsso.org/Documents/2008/Learning_Progressions_Supporting_2008.pdf.

Lewis, C. (Producer). (2005). How many seats? [DVD]. Lesson Study Group at Mills College. Ordering information retrieved September 2014 http://www.lessonresearch.net/ordering1.html.

Love, N. B., Stiles, K. E., Mundry, S.E., & DiRanna, K. (Editors). (2008). *The data coach's guide to improving learning for all students: Unleashing the power of collaborative inquiry.* Thousand Oaks, CA.: Corwin Press.

National Council of Supervisors of Mathematics. (2008). *The PRIME leadership framework: Principles and indicators for mathematics education leaders*. Denver, CO: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics (PSSM).* Reston, VA: Author.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics.* Washington, DC: Authors.

Romagnano, L. (2006). *Mathematics assessment literacy: Concepts and terms in large-scale assessment.* Reston, VA.: National Council of Teachers of Mathematics.

Wang, P & Yoshida, M. (2005). *Building our understanding of lesson study.* Philadelphia, PA, Research for Better Schools.

Student Supplementary Readings

Grant, C.M. & Davenport, L.R. (2009). Principals in partnership with math coaches. *Principal, 88(*5), 36-41.

Hodges, T., Rose, R, & Hicks, A. (2012). Interviews as RtI Tools. *Teaching Children Mathematics*, *19*(1) 30-36.

Killion, J. (2010). Reprising coaching heavy and coaching light. *Learning Forward*, *6*(4), 8-9.

Knight, J. (2009). What can we do about teacher resistance? *Phi Delta Kappan*,90(7), 508-513.

Knight, J. (2011). What good coaches do. *Educational leadership*, 69(2), 18-22.

Lipton, L., & Wellman, B. (2007). How to talk so teachers listen. *Educational leadership*, Association for Supervision and Curriculum Development.

Lewis, C., Perry, R. & Murata, A. (2006). How should research contribute to instructional improvement? A case of lesson study. *Educational Researcher*, 35(3), pp. 3-14.

National Staff Development Council (2006) . NSDC tool: Developing a partnership agreement between a coach and a principal 2(4), 8-9.

Stein, M. K., Remillard, J. T., & Smith, M. (2007). How curriculum influences student learning. In F. K. Lester (Ed.), Second handbook of research on mathematics teaching and learning (pp. 319-370). Charlotte, NC: Information Age Publishing.

Relevant position papers developed by the National Council of Teachers of Mathematics and the National Council of Supervisors of Mathematics: Leadership in Mathematics Education.

Websites: Videos and Other Resources

Dylan Wiliam, http://www.dylanwiliam.org/Dylan_Wiliams_website/Welcome.html. A collection of materials developed by Wiliam based on his research and work in assessment.

Inside Math Project, http://www.insidemathematics.org/. Provides videos of K-12 exemplary lessons being taught using the 8 mathematical practices spelled out in the Common Core State Standards as well as specific content standards at a variety of grade levels. Aligns well with Virginia Standards of Learning. The mathematical practices align with Virginia's Process Goals.

The Mathematics Assessment Project, http://map.mathshell.org/materials/index.php. The Mathematics Assessment Program (MAP) aims to bring to life the Common Core State Standards (CCSSM). MAP is a collaboration between the University of California, Berkeley and the Shell Center team at the University of Nottingham, with support from the Bill & Melinda Gates Foundation. The team works with the Silicon Valley Mathematics Initiative and school systems across the US and the UK to develop improved assessments.

The Teaching Channel, https://www.teachingchannel.org/. Videos of K-12 exemplary lessons being taught using the mathematical practices and specific Common Core Content Standards at a variety of grade levels. Aligns well with Virginia Standards of Learning. The mathematical practices align with Virginia's Process Goals.

Virginia Department of Education (VDOE) Professional Development Resources for Mathematics, http://www.doe.virginia.gov/instruction/mathematics/professional_development/index.shtml. A collection of professional materials.

Leadership III Course Outline: Topics and Essential Questions

The outline presented below shows the course scheduled as **8 six-hour classes.** The essential questions provided in the second column guide the selection of topics as well as the development of activities and resources identified in column 3. Column 3 shares a sampling of activities, readings, and assignments for each class as well as when major projects are assigned and due. Textbook assignments are identified in column 3 using the code below.

- **EMSM** -- *Elementary and Middle School Mathematics Teaching Developmentally Seventh Edition*
- ***HEMS*** -- *Handbook for Elementary Mathematics Specialists*
- **UD --** *Using Data to Improve Learning for All: A Collaborative Inquiry Approach*
- **ISBMI** -- *Implementing Standards-based Mathematics Instruction: A Case for Professional Development, Second Edition*
- **EFA** -- *Embedded Formative Assessment*

Figure 11. Leadership III course: Overview of Topics

Class	Essential Questions	Activities and Suggested Resources
1	What characteristics define a team and what attributes are important to be an effective team member or team leader? Why is assessment important and what does it mean for a mathematics specialist to be assessment literate? What does it mean to use data to improve learning? How can the Task Analysis Guide support thinking about cognitive demand and what constitutes a "worthwhile" mathematics task? What can teachers learn about their students when teaching through problem solving?	Pyramid Cup Stacking Activity and discussion- Adapted for 15 cups from http://4h.ucanr.edu/files/121325.pdf Interactive reading and discussion of UD Chapter 1, Building a High-Performance Data Culture. Table groups have 10 statements about assessment and the use of assessments to determine if T or F and what thinking lead to their decision. A whole class discussion follows that includes important ideas from the pre-class reading, EFA Chapter 1, Why Educational Achievement Matters. Use the NCTM MTMS article, Thinking Through a Lesson: Successfully Implementing High-level Tasks. Have students engage in the marble task before reading the article Use the marble task and page 6 in ISBMI to revisit the four levels of cognitive demand and the Task Analysis Guide which were introduced during

	What does one need to look for in a professional book to inform the work of a mathematics leader?	Leadership I. Reading, interactive activity, and discussion using the NCTM 2010 Problem Solving Research Brief: Why Is Teaching With Problem Solving Important to Student Learning? http://www.nctm.org/news/content.aspx?id=25713. Examine the 10 worthwhile-problem criteria thoroughly and compare to the indicators in the Task Analysis Guide and to the NCTM Process Goals. *Project Overview and Guidelines:* • Professional Book Review (Due Class 4) *Homework:* • Reflection Journal Prompt based on reading the NCTM MTMS article, Thinking Through a Lesson: Successfully Implementing High-level Tasks. • Read EFA Chapter 1, Why Education Achievement Matters and Chapter 2, The Case for Formative Assessment
2	What is formative assessment and why is it important for teachers and students? How can mathematics specialists support teachers and administrators to develop and use formative assessments as an integral part of the school's assessment program? What is the role of learning progressions and learning targets in formative assessment? Prepare for Student Diagnostic Interview Project. (Participants have planned and conducted student interviews in previous leadership courses.) What knowledge and skills support mathematics specialist in helping	Interactive group activity to develop a shared understanding of 1) what is formative assessment, 2) what is the purpose and value of using formative assessments, and 3) what beliefs and knowledge are necessary for teachers to make formative assessment an effective component of the teacher's practice. Small group and whole group discussions will incorporate information from EFA Chapter 2, The Case for Formative Assessment. Use an interactive small group activity to revisit the concept of learning targets and learning progressions which were introduced in Leadership I. Provide a reading guide and class time for students to read EFA Chapter 3, Clarifying, Sharing, and Understanding Learning Intentions and Success Criteria.

	teachers incorporate mathematics learning progressions and learning targets in planning instruction? What resources are available at the state of education website to support a mathematic specialist's work to provide leadership for improving the school's mathematics program, classroom teaching and assessment, and student achievement. What is an achievement gap and what factors may impact creating or closing an achievement gap?	Develop a small group activity to have participants process the information in EFA Chapter 3. The activity will develop their understanding of the role of learning progressions and learning targets in developing formative assessments and in interpreting the resulting student learning data for instructional planning. Develop an in class website search for pairs to learn about and catalog for later use the various resources on the state department of education website. Information on the website will be critical in the Decision Making with School Data Project. Prepare students with the background information, including the state's definition of a proficiency gap, necessary to carry out the Working with Proficiency Gap Sub-groups Project. Review the project guidelines and have the group brainstorm what questions they need to answer to complete the project and what resources they already may know about that may be helpful. Bring out the information and resources available at the state department of education website. *Project Overview and Guidelines:* • Working with Proficiency Gap Sub-groups Project (Due Class 5) • Student Diagnostic Interview Project (Due Class 3) *Homework:* • Reflection Journal Prompt based on reading HEMS Chapter 7, Supporting Teachers' Work with Special Education Students and Chapter 8, Supporting Teachers Work with English Language Learners and Gifted Mathematics Learners. • Read UD Chapter 4, Bringing Cultural Proficiency to Collaborative Inquiry and EMSM Chapter 6, Teaching Mathematics Equitably to All Children to inform the Working with Proficiency Gap Sub-groups Project.

		• Read NCTM TCM article, Interviews as RtI Tools to inform planning the diagnostic student interview. • Do the mathematics on page 57 of Chapter 6, Multiplying Fractions with Pattern Blocks, in the ISMI book. Then read the case of Fran Gorman and Kevin Cooper on pages 56-63. Thank about the ideas developed in our first two classes and compare Fran's lesson to Kevin's lesson and be ready to use your comparisons in the next class.
3	What can be learned from a student interview as a formative assessment that supports improving student learning? What formative assessment strategies serve to make student learning visible? What strategies can be used to guide the modification of textbook tasks to increase rigor and strengthen student understanding and to make student learning visible? What supports do teachers need as they gain confidence and abilities to implement worthwhile tasks in a way to maintain the rigor and as formative assessment? What are is the value and purpose of the components in the lesson study framework? What is the role of lesson study in a professional learning community working to improve student learning?	Debrief Student Diagnostic Interview Project Interactive small group activities and discussions using EFA Chapter 4, Eliciting Evidence of Learners' Achievement and EMSM chapter 5, Building Assessment into Instruction. Interactive activity to learn how to revise textbook tasks to increase rigor and the level of student thinking. Activity developed using the 2013 VDOE professional development resources, http://www.doe.virginia.gov/instruction/ mathematics/professional_development/index.shtml. and MAP resources, http://map.mathshell.org/materials/index.php. Form expert groups to jigsaw discuss the information in the assigned readings. • ISMI Chapter 11, Assisting the Learning of Teachers • Merritt, E., Rimm-Kaufman, S., Berry, R., Walkowiak, T. & McCracken, E. (2010) . A Reflection framework for teaching math (MSCANS). *Teaching Children Mathematics.,17*(4). Provide focus questions that will support a jig-saw group discussion about the supports teachers need to develop their capacity to effectively implement worthwhile tasks. The jig-saw group discussions end by looking back at the Gorman and Cooper case in ISMI Chapter 6 and developing ways that a mathematics specialists can support both of the teachers in the caseas they continue to work on using tasks in their classrooms.

<table>
<tr><td></td><td></td><td>Introduce lesson study by asking each student to use 3 post-it notes to write 3 words or short phrases they associate with lesson study. Ask the table groups to share their post-it notes and to develop umbrella categories for clustering the post-its. Have table groups share.

Homework:
Provide a Reflection Journal Prompt based on one or more of the ideas address addressed so far that will encourage participants to reflect and strengthen their understanding.
To develop an understanding of the lesson study as a framework for improving teaching and learning watch or read the following and be ready to use the information in the next class.
- Watch the SEPTEMBER 29, 2010, 6:30 PM CBS News video: Catherine Lewis, distinguished research fellow of education at Mills College, speaks about how Japanese teachers learn by working with others. <u>http://www.cbsnews.com/video/watch/?id=6912687n#ixzz1ZlqbuJCA</u>
- Read the article <u>A Deeper Look at Lesson Study</u> Lewis, C., Perry, R. & Hurd, J. (2004). Educational Leadership. February 2004, pp.18-22 at http://www.lessonresearch.net/newtols.html

- Read EMSH, Chapter 5, pages 80-84, Leading Lesson Study</td></tr>
<tr><td>4</td><td>What are the stages of lesson study and how do they support improving instruction and student learning?

What knowledge and skills are necessary for mathematics specialists to support teachers in lesson study?

What are the next steps in planning</td><td>Professional Book Review project presentations. View and discuss the DVD, How Many Seats? and provide note-taking forms based on the stages of lesson study. Stop after each of the 5 video segments to discuss what actions of the planning team, teacher, students, and observers in the public lesson, and the participants in the debrief.

Debrief the homework readings about lessons student to make explicit the stages of lessons study and the knowledge and skills necessary to support teachers in</td></tr>
</table>

	for the Lesson Study Project? What knowledge and skills support effective clinical classroom observations for both the mathematics teacher leader and the classroom teacher?	lesson study. Develop a small group activity to develop an understanding the necessary steps in planning and writing a lesson plan for a lesson study. This activity and discussion will bring out the importance of having a clear learning target, examining the learning progression for the concept, identifying and implementing a rich task, and strategies for formative assessment. Review in detail the Lesson Study Project Guidelines. *Project Overview and Guidelines:* • *Detailed review of* Lesson Study Project (Due Class 8) *Homework:* • *Journal Prompt:* Read: Lewis et al. (2006). How Should Research Contribute to Instructional Improvement? A Case of Lesson Study. Based on this article, describe the components of Lesson Study and how they contribute to instructional improvement. • Read EMSH Chapter 4, Supporting Grade Level teachers to inform the Lesson Study Project. • Refer back to IMSI Chapter 11, page 136 Thinking Through a Lesson Protocol to inform the Lesson Study Project. • Read EFA Chapter 5, Providing Feedback that Moves Learning Forward. Highlight 5 sentences that resonate with you and be ready to discuss the chapter in the next class.
5	What knowledge and skills are necessary for mathematics specialist to provide leadership for the school's mathematics program and to support teachers in closing the achievement gap? When using formative assessment effectively, what are the critical features of feedback that will move learning forward?	Working with Proficiency Gap Sub-groups Project Presentations and follow-up discussion. Following the presentations, form mixed groups so that each sub-group is represented. As the small groups to discuss how what each sub-group researcher learned about how ways to support the various sub-groups were similar and different. In a whole group discussion chart the similarities and differences and how school programs and classroom instruction can support closing the achievement gap while raising rigor for all students. Use an interactive activity and discussion to debrief

	How can classroom observations inform the work of a mathematics specialist or teacher leader and be used as a tool to improve learning for all students?	EFA Chapter 5, Providing Feedback that Moves Learning Forward. Use the professional development video, task, and sample student work from the Math Assessment Project, Formative Assessment How can I respond to students in ways that improve their learning? found at http://map.mathshell.org/static/draft/pd/modules/1_Formative_Assessment/pdf/1_Formative_Assessment_Guide.pdf to create a simulation for participants. Prepare for the Classroom Diagnostic Observation Project using a video clip of a classroom lesson and the classroom observation guides found in Grant et al. (2006). *Lenses on learning supervision: Focusing on mathematical thinking, Facilitator book* simulates an observation cycle; pre-conference, observation, and post-conference. *Project Overview and Guidelines:* • Classroom Diagnostic Observation Project (Due Class 6) *Homework:* • Journal Prompt: Read EMSH Chapter 9, Using Assessment to Inform Instruction on the Basis of Data. An important role of a mathematics specialist is to support teachers in their work to improve student learning. Write a 2-page memo to your principal sharing the five stages in the Using Data Process and how you think each can play a role in increasing student achievement in mathematics at your school. • Read the UD Chapter 3, The Using Data Process. pages 45-55; 60-79 and respond to the focus questions provided by the instructor.
6	How can classroom observations inform the work of a mathematics specialist or teacher leader and be used as a tool to improve learning for all students?	Debrief Classroom Diagnostic Observation Project and discuss how classroom observations can support the work of the mathematics specialist. Note: Instructors will find the professional development guide developed by Love et al. (2008),

	What knowledge and skills enable the use of various levels of data to improve student learning? What are appropriate uses for the different types of student-learning data and why is the drill-down process necessary for effective instructional decision making? How do the 4-phases of the data dialogue discussion support an object examination of data to support identifying student learning problems?	*The Data Coach's Guide To Improving Learning For All Students: Unleashing The Power Of Collaborative Inquiry*, helpful in planning activities based on the UD book. Develop a small group activity using UD Chapter 3, The Using Data Process, pages 55-60 and the data samples in Table 3.1, Drilling Down into multiple Levels of Student-Learning Data to investigate what can be learned from the different levels of data? How does each level of data support the work of the mathematics specialist to improve student learning? The instructor will supply samples of data for a simulation of the 4-phases of the data dialogue process described in UD Chapter 3, The Using Data Process: A Model for Collaborative Inquiry, pages 60-61. .Introduce the stoplight highlighting protocol as a process for Data-Driven Dialogue, making observations and inferences about the highlighted data that enables a group to highlight student learning needs and successes. Set up a small group activity and simulate analyzing assessment data reports using the stoplight highlighting protocol. *Project Overview and Guidelines:* • Decision-Making with School Data Project (Due Class 8) *Homework: TBD*
7	What are the important steps that enable identifying student learning problems and developing SMART goals to address the learning problem? How can partnerships with teachers, administrators, and other school staff support the work of the mathematics specialist? What does it mean to be a	Use UD Chapter 3, The Using Data Process: A Model for Collaborative Inquiry, pages 61-74 to explore Identifying a student learning problem, writing a student learning problem statement, and writing a SMART Goal.

	change agent when serving in the role of a mathematics specialists? *At this point, instructors reflect on the class goals and on students current understanding to identify ideas to revisit or extend.*	
8	How can lesson study be used to support improvement in instruction and student learning? What knowledge and skills do school-based mathematics leaders need to facilitate data discussions to inform instruction? How can self-reflection support the growth of a school-based mathematics leader?	Discussion of the Lesson Study project to have the group reflect on the value of the lesson study process as a professional development strategy, and what is learned through the lesson design process and the public teaching of the lesson that can be carried into the work of a mathematics specialist? Debrief Decision Making with School Data Project and simulation of having data conversations, data-driven dialogue as described on pages 60-61. Debriefing activity to reflect across the three leadership courses that will enable future specialists to think about how they will carry out their role during the first year. What materials do they want to keep ready at hand to do their work? It may be helpful to have some vignettes of challenges of a mathematics specialist for some discussion. *Project Overview and Guidelines:* Course Exam: Final Personal Growth Reflection Paper (Due 1 week from date Class 8)

Leadership III Class 2 Project:
Proficiency Gap Sub-Groups Quick Reference Guide

The "achievement gap" in education refers to the disparity in academic performance between groups of students. The achievement gap shows up in grades, standardized test scores, course selection, dropout rates, and college completion rates, among other success measures. The achievement gap, as it is usually reported, is NOT the difference in achievement test scores between two groups of students. Rather, it is the difference between the percentages of students that are proficient in each sub-group.

Virginia proficiency gap groups. The Virginia Department of Education considers the reading and mathematics performance of students in three "proficiency gap groups" comprising students who historically have had difficulty meeting the state's achievement standards. The student subgroups are used to identify Focus schools under Virginia's 2012 flexibility waiver.

- **Proficiency Gap Group 1** – Students with disabilities, English language learners and economically disadvantaged students, regardless of race and ethnicity
- **Proficiency Gap Group 2** – African-American students, not of Hispanic origin, including those also counted in Proficiency Gap Group 1
- **Proficiency Gap Group 3** – Hispanic students, of one or more races, including those also counted in Proficiency Gap Group 1

The proficiency gap sub-groups project. This project will enable you to learn more about specific groups of students assigned to particular Proficiency Gap Groups and how to support the mathematics learning for each category of students.

1. Working in groups of 4 sign up for one of the specific student populations identified below. identified below.

    A. Proficiency Gap Group 1 – Students with disabilities
    B. Proficiency Gap Group1- English language learners
    C. Proficiency Gap Group1- Economically disadvantaged students
    D. Proficiency Gap Group 2 – African-American students
    E. Proficiency Gap Group 3 – Hispanic students

2. In order to learn more about your selected specific student populations, the group will conduct a literature review to learn more about the following.
    a. What are the potential challenges the particular group of students faces when learning mathematics?
    b. What are research-based best practices for supporting the specific groups of students in learning mathematics?
    c. What is important for teachers to know about working with the particular group of students and their families?

3. The product for this project is to develop a "quick reference guide" to share with other members of the class that can be used as a tool for mathematics specialists and teachers to use in their daily work. The information in the guide will make the reader or user aware of potential instructional and learning challenges and provide suggestions for ways to help these students improve their mathematics learning. The guide can take any format your group wishes but must be research-based, easy to follow, and provide helpful information.

4. Each group will provide a 20-minute presentation about their student populations and share their Quick Reference Guide. The presentation should inform the class about the potential instructional and learning challenges and provide suggestions for ways to help these students improve their mathematics learning. You may have a PowerPoint, but it is not required. Provide either a hard copy or send an electronic copy or your Quick Reference Guide to your classmates.

Rubric for *Proficiency Gap Sub-Groups Project*

Exceptional (95- 100%)	**Proficient**	**Accomplished**	**Competent**	**Does Not Meet Expectations**
Presentation of Written Information in the Quick Reference Guide **/20 Points**	(20-16 Points) Very well written; a logical format that was easy to follow. Flowed smoothly from one idea to another. Transitions were easy to follow. No issues with writing mechanics.	(16 – 12 Points) Well written; a logical format that was easy to follow. For the most part, flowed smoothly from one idea to another. Most transitions cold be followed fairly easily. No more than 2 issues with writing mechanics.	(11 – 7 Points) Clearly written, but at times ideas were not presented coherently. Some parts of the information lacked a smooth flow from one idea to another. Some transitions were lacking in helping the reader follow the ideas being presented. No more than 4 issues with writing mechanics.	(6-1 Points) Choppy and confusing; the format was difficult to follow transitions of ideas were abrupt and seriously distracted the audience. More than 4 issues with writing mechanics.
	(20-16 Points) Provided a well	(16 – 12 Points) Provided an	(11 – 7 Points) Provided a less than	(6-1 Points) Provided a less

Content **/20 Points**	thought out and complete response for each of the three questions in #2. All information was concise, explicit, and helpful for mathematics instruction for the sub-population.	adequate response to all three questions in #2. All information was clear, accurate, and helpful for mathematics instruction for the sub-population.	adequate response to no more than 1 question in #2. Some but not all information was clear, accurate, and helpful for mathematics instruction for the sub-population.	than adequate response to more than 1 question in #2. Information was lacked clarity, accuracy, or appropriate help for the subpopulation.
Research **/20 Points**	(20-16 Points) Used at least 6 sources of level 1 and 2 resources to provide much information to address the three questions in #2 and to help teachers and mathematics leaders.	(16 – 12 Points) Used at least 5 sources of level 1 and 2 resources to gather helpful information to address the three questions in #2 and to help teachers and mathematics leaders.	(11 – 7 Points) Used at least 4 sources of level 2 resources to gather information. Some information may not have directly addressed the three questions in number 2.	(6-1 Points) Used no more than 3 sources of level 2 resources to gather information. Some information may not have directly addressed the three questions in number 2.
The Product: The Quick Reference Guide **/20 Points**	(20-16 Points) The focus population was clearly identifiable, and information was organized for ease of use. The product “looked” professional and worthy of sharing with administrators and other	(16 – 12 Points) The focus population could be identified with little effort, and there was an organizational structure for the information. The product, for the most part, “looked” professional and worthy of sharing with administrators	(11 – 7 Points) The focus population could be identified with some effort, and there was an apparent attempt at an organizational structure for the information. The product did not consistently have the look of a professional product that invited sharing with	(6-1 Points) The focus population was not clearly identified. The product lacks an organizational structure. The product did not have the look of a professional product that invited sharing with administrators and other educators.

	educators.	and other educators.	administrators and other educators.	
Group Oral Presentation **/10 Points**	(10 – 8) Presentation is well organized with a beginning, middle, and end. There is a strong organizing theme, with clear main ideas and transitions.	(7-5) Speaker loses train of thought, does not stay with the proposed outline, or connections are attempted but not made clear for the audience.	(4-1) Presentation shows little organization, unclear purpose, and/or unclear relationships or transitions.	
Team Work **/10 Points**	Each team member will evaluate contributions of the other members of the team.			

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Section 4:

VMSI Research and Publications Bibliography

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VMSI Research and Publications Bibliography

As described in this report, the VMSI is a multifaceted collaborative effort involving more than 50 school systems, 10 universities, the VDOE, three Virginia mathematics education organizations, and the VMSC. During the past two decades much has been learned in Virginia about preparing mathematics specialists to coach teachers and to provide leadership for a school's mathematics program. Emerging research from the Virginia grant projects suggests that well-prepared specialists working with classroom teachers do have an impact on student learning (Campbell & Malkus, 2011). Furthermore, "Mathematics Specialists are now widely known and well regarded in the public schools and their communities" (Blount & Singleton, 2013, p. 253).

But, much work remains in Virginia to sustain and advance preparing and launching mathematics specialists. For example, the institutionalization of the mathematics specialist role in elementary and middle schools is dependent on securing financial support to 1) prepare adequate numbers of specialists and 2) fund school-based specialists to serve in all elementary and middle schools. In addition, to strengthen principals and central office leaders understanding their role in supporting mathematics specialists to improve student learning by working with teachers.

The initiative, with grant support, produced a substantial collection of products. For convenience, the research and publications that follow have been identified in three categories: research publications, preparing and supporting mathematics specialists, and supporting principals. Articles from some journals and books may be acquired at a cost. However, many materials are freely accessed online.

Research Publications

Blount, D. & Singleton, J. (2007). The role and impact of the mathematics specialist from the principals' perspectives. *The Journal of Mathematics and Science: Collaborative Explorations*, *9*, 69-77. Retrieved from http://www.math.vcu.edu/g1/journal/Journal9/JournalWork/FormattedCopies/Blount-Singleton.html.

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Whitenack, J., & Ellington, A. (2007). A Methodology to Explain Teachers' Emerging Roles as K-5 Mathematics Specialists. Paper presentation to be given at the annual meeting of the American Educational Research Association, Chicago, IL. Retrieved at http://www.vamsc.org/Impact/Whitenack_Ellington_AERA_2007.pdf.

Whitenack, J., & Ellington, A. (2009). K-5 mathematics specialists' teaching and learning about fractions. *The Journal of Mathematics and Science: Collaborative Explorations*, *11*, 109 – 126. Retrieved at http://math.vcu.edu/outreach/the-journal-of-mathematics-and-science-collaborative-explorations/journal-issue-volume-11-spring-2009.

Whitenack, J. W., Cavey, L. O., & Ellington, A. J. (2009). The instructor's important role in supporting mathematical arguments in a k-5 mathematics specialist program. Conference Proceedings of the 12th Conference on Research in Undergraduate Mathematics Education. Retrieved at http://sigmaa.maa.org/rume/crume2009/Whitenack_LONG.pdf.

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